Error Estimate of Bending Angles in the Presence of Strong Horizontal Gradients

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Abstract The CT/FSI (Canonical Transform/Full-Spectrum Inversion) technique permits achieving a high accuracy and vertical resolution in the retrieval of bending angle from radio occultation data. This technique can be universally applied for the (hypothetical) spherically-symmetric atmosphere and any multipath situation can be unfolded. The reason is that the CT/FSI technique uses a Fourier Integral Operator that maps the measured wave field into the impact parameter representation, and for a spherically-symmetric medium each ray has a unique impact parameter. For the real atmosphere with horizontal gradients the situation is different. Horizontal gradients result in the variation of the impact parameter along a ray. In the presence of strong horizontal gradients, a bending angle profile can become a multi-valued function. In this case, the CT/FSI technique in its standard variant will fail to correctly retrieve the bending angle profile. It is, however, possible to estimate bending angle errors. For this purpose we apply the sliding spectral analysis of the CT-transformed wave field. The spectral width is used as a measure of the bending angle errors. We perform numerical simulations with global fields from re-analyses of the European Centre for Medium-Range Weather Forecasts and show that this radio holographic technique can be effectively used for error estimation in the areas of multi-valued bending angle profiles.

1 Introduction

The Canonical Transform (CT) (Gorbunov 2002; Gorbunov and Lauritsen 2004), Full-Spectrum Inversion (FSI) (Jensen et al. 2003), and Phase Matching (Jensen et al. 2004) methods were designed for the reconstruction of the ray manifold structure from the measurements of the complex wave. They are widely used for the retrieval of bending angle profiles from radio occultation (RO) data. The central concept of the CT method is the ray manifold in the phase space. The canonical coordinates (coordinates and momenta) in phase space can be chosen in different ways. A particular choice is the physical coordinate and ray direction vector projection to the coordinate axis. This coordinate system is used for the description of the physical wave field. Multipath propagation corresponds to the multi-valued projection of the ray manifold to the coordinate axis. For the retrieval of the ray manifold structure it is however necessary to find another coordinate axis such that the ray manifold should have a single-valued projection upon it. For the rays in a spherically-symmetrical atmosphere the impact parameter is an invariant quantity that is constant for each ray. The impact parameter is, therefore, a unique coordinate along the ray manifold. Impact parameter and bending angle are conjugated coordinate and momentum. The canonical transform from the physical coordinate and ray direction vector projection to impact parameter and bending angle would then completely disentangle multipath structure. The impact parameter provides a universal coordinate choice for the case of the spherically symmetrical atmosphere.

The situation changes as we consider the atmosphere with horizontal gradients. In this case, it is possible to introduce the effective impact parameter, whose definition will depend on the horizontal gradients of refractivity. It turns out that the standard CT algorithm can work in most practical situations. However, numerical simulations with global fields from the European Centre for Medium-Range Weather Forecasts (ECMWF) do also reveal cases where atmospheric horizontal gradients are strong enough to make the bending angle a multi-valued function of the effective impact parameter. Because the structure of the ray manifold depends on the unknown horizontal gradients, it proves impossible to specify a universal coordinate choice that can unfold multipath. Therefore, it is necessary to estimate bending angle errors.

Two approaches were introduced for the dynamic estimate of bending angle errors, both based on the analysis of the CT/FSI-transformed wave field: 1) the sliding spectral analysis of the full complex wave fields in the transformed space (Gorbunov et al. 2005, 2006) and 2) the analysis of the fluctuation of the amplitude of the wave field in the transformed space (Lohmann 2006). The first approach is applied in the operational processing of RO data. The second approach was recently used to estimate the summary effect of receiver tracking errors and lower-tropospheric turbulence and to generate maps of convection and turbulence structures (Sokolovskiy et al. 2007).

Here, we estimate errors of the bending angle retrieval by using the sliding-spectral analysis of the CT-transformed wave field. We will present some atmosphere examples with horizontal gradients and obtain results for the corresponding bending angle error estimates.

2 Ray Manifold and its Description in the Phase Space

In a RO experiment rays are emitted by a GPS satellite, pass through the atmosphere, where they undergo refraction, and are received by a Low-Earth Orbiter (LEO). Each ray may be characterized by its impact parameter p. For a spherically symmetrical medium, Snell's law reads (Kravtsov and Orlov 1990):

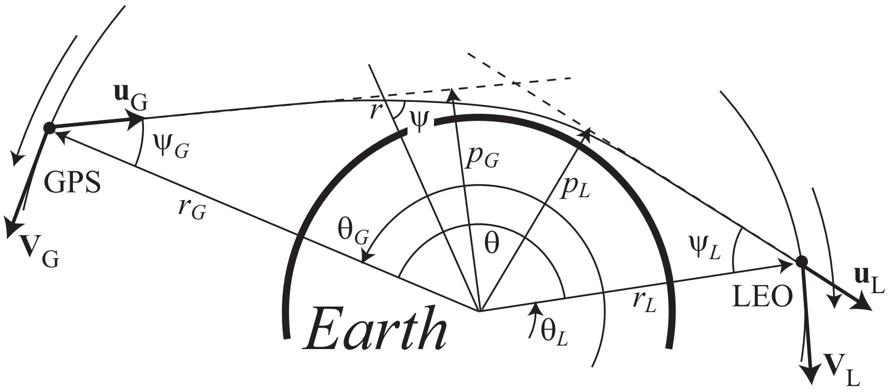

Fig. 1 Schematic drawing of the occultation geometry. The various quantities are discussed in the main text.

$$nr\sin\psi = p = \text{const} \tag{1}$$

where ψ is the angle between the ray direction and the radius vector r (Fig. 1) and n is the refractive index. Since the GPS and LEO satellites are located outside of the atmosphere where $n = 1$ (here we neglect the ionosphere), it follows that $p = r_{\mathrm{G}}\sin\psi_{\mathrm{G}} = r_{\mathrm{L}}\sin\psi_{\mathrm{L}}$, which equals the leveling distance between the ray and the Earth's curvature center.

When considering an atmosphere with horizontal gradients, the situation changes. The complex wave field in the inhomogeneous medium can be written in the following form: $u(\mathbf{x}) = A(\mathbf{x})\exp(ik\Psi(\mathbf{x}))$, where $\mathbf{x} = (x^i)$ is the coordinate vector, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, $A(\mathbf{x})$ is the amplitude, and $\Psi(\mathbf{x})$ is the eikonal. The geometrical optics of inhomogeneous media is based on the eikonal equation (Kravtsov and Orlov 1990):

$$(\nabla\Psi)^2 = n^2(\mathbf{x}) \tag{2}$$

Rays are described by the Hamilton system, where the Hamilton function follows from Eq. (2): $H(\mathbf{p},\mathbf{x}) = 1/2\cdot(\mathbf{p}^2 - n^2(\mathbf{x}))$, where $\mathbf{p} = \nabla\Psi$ is the momentum. The associated Hamilton equations for a ray take the following form:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\tau} = \mathbf{p}; \qquad \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}\tau} = n\nabla n, \tag{3}$$

where τ is the trajectory parameter. Along the ray trajectories, $H(\mathbf{p},\mathbf{x}) = 0$, therefore $|\mathrm{d}\mathbf{x}/\mathrm{d}\tau| = |\mathbf{p}| = n(\mathbf{x})$, which allows for the conclusion that $\mathrm{d}\tau = \mathrm{d}s/n$ (since $\mathrm{d}s = |\mathrm{d}\mathbf{x}|$). Consider now polar coordinates (r,θ) and the corresponding metrics $\mathrm{diag}(1, r^2)$. The equations for the angular component of the momentum p_θ take the following form:

$$\begin{aligned} p_\theta &= r^2\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = nr\frac{r\mathrm{d}\theta}{\mathrm{d}s} = nr\sin\psi; \\ \frac{\mathrm{d}p_\theta}{\mathrm{d}\tau} &= n\frac{\partial n}{\partial\theta} \end{aligned} \tag{4}$$

From the first equation we see that in the polar coordinates the angular component of momentum equals the ray impact parameter as it is defined for a spherically symmetrical medium, i.e., $p = p_\theta$. The second equation generalizes Snell's law (Eq. 1) and indicates that p is only invariant in a spherically symmetrical medium $(\partial n/\partial\theta = 0)$. When discussing the RO sounding of the atmosphere with horizontal gradients we must consider two impact parameters: $p_\mathrm{G} = r_\mathrm{G}\sin\psi_\mathrm{G}$ and $p_\mathrm{L} = r_\mathrm{L}\sin\psi_\mathrm{L}$. The relation between them follows from Eq. (4):

$$p_\mathrm{L} = p_\mathrm{G} + \int_\mathrm{GPS}^\mathrm{LEO}\frac{\partial n}{\partial\theta}\mathrm{d}s. \tag{5}$$

In processing radio occultation data it is not possible to directly determine the two impact parameters, at the receiver and at the transmitter. Instead, an effective impact parameter can be found. It is computed from σ, the time derivative of the optical path $\Psi(t)$ measured along the LEO observation trajectory as a function of time t, using the same formulas as for the unique impact parameter in a spherically symmetrical atmosphere. The general expression for σ is as follows:

$$\begin{aligned} \sigma &= \frac{\mathrm{d}\Psi}{\mathrm{d}t} = \mathbf{V}_\mathrm{L}\cdot\mathbf{u}_\mathrm{L} - \mathbf{V}_\mathrm{G}\cdot\mathbf{u}_\mathrm{G} \\ &= \dot{r}_\mathrm{G}\cos\psi_\mathrm{G} + r_\mathrm{G}\dot{\theta}_\mathrm{G}\sin\psi_\mathrm{G} + \dot{r}_\mathrm{L}\cos\psi_\mathrm{L} - r_\mathrm{L}\dot{\theta}_\mathrm{L}\sin\psi_\mathrm{L} \\ &= \frac{\dot{r}_\mathrm{G}}{r_\mathrm{G}}\sqrt{r_\mathrm{G}^2 - p_\mathrm{G}^2} + \frac{\dot{r}_\mathrm{L}}{r_\mathrm{L}}\sqrt{r_\mathrm{L}^2 - p_\mathrm{L}^2} + p_\mathrm{G}\dot{\theta}_\mathrm{G} - p_\mathrm{L}\dot{\theta}_\mathrm{L}. \end{aligned} \tag{6}$$

Here, in the final expression for σ we neglect horizontal gradients perpendicular to the occultation plane. For the effective impact parameter p we have the following implicit definition:

$$\sigma = p\dot{\theta} + \frac{\dot{r}_\mathrm{G}}{r_\mathrm{G}}\sqrt{r_\mathrm{G}^2 - p^2} + \frac{\dot{r}_\mathrm{L}}{r_\mathrm{L}}\sqrt{r_\mathrm{L}^2 - p^2}. \tag{7}$$

Using the following formula for the optical path:

$$\Psi = |\mathbf{r}_\mathrm{L} - \mathbf{r}_\mathrm{G}| + \Delta S, \tag{8}$$

where ΔS is the phase excess, we can obtain the effective impact parameter from orbit data and phase excess by numerically solving Eq. (7) in single ray areas, or by applying the Canonical Transform (CT) (Gorbunov 2002; Gorbunov and Lauritsen 2004), Full-Spectrum Inversion (FSI) (Jensen et al. 2003), or Phase Matching (Jensen et al. 2004) methods for unfolding multipath propagation. These techniques are based on the assumption that the effective impact parameter as defined by Eq. (7) is a unique coordinate of the ray manifold, in other words, that different rays have different p's. The (implicit) relation between p_{G}, p_{L}, and p is given by Eqs. (5), (6), and (7). It includes the horizontal gradient of refractivity $\partial n/\partial\theta$.

In the presence of strong horizontal gradients varying with height the effective impact parameter may no longer represent a unique coordinate of the ray manifold. One possible way of dealing with this situation would be to modify the CT method in such a way that the ray manifold is projected to a coordinate different than p. By itself, this might be straightforward except for the fact that this new unique coordinate is unknown, because it depends on the unknown horizontal gradients of refractivity. Here we will follow another prospect: instead of modifying the CT method, we will estimate bending angle errors due to strong horizontal gradients.

3 Radio Holographic Error Estimation

Radio holographic analysis of the wave signal can be performed both in t- and p-domains. The radio holographic (or sliding spectrum) analysis in the t-domain is widely used for the visual analysis of RO data in order to identify reflected rays, different sorts of problems etc. (Igarashi et al. 2000; Gorbunov et al. 2005). The radio holographic analysis in p-domain allows for the error estimate of retrieved bending angles (Gorbunov et al. 2005).

Here we will apply the method for the estimate of the bending angle error due to non-fully unfolded multipath structure (Fig. 2).

In the CT method, the wave field $u(t)$ is mapped to the impact parameter representation by the Fourier Integral Operator (FIO) $\hat{\Phi}$. The field in the transformed space, $\hat{\Phi}u(p) = A'(p)\exp(ik\Psi'(p))$ can be subjected to the sliding spectral analysis (Gorbunov et al. 2005):

$$w(p,\xi) = \int\limits_{p-\Delta p/2}^{p+\Delta p/2} \cos\frac{\pi(p'-p)}{\Delta p}\,\frac{\hat{\Phi}u(p')}{\exp\left(ik\overline{\Psi}'(p)\right)}\exp\left(-ik\xi p'\right)\mathrm{d}p', \tag{9}$$

where Δp is the window width, $\overline{\Psi}'(p)$ is the model of the phase variation in the p-domain, and ξ is the bending angle variation. $w(p,\xi)$ has a maximum near the true bending angle $\xi = \varepsilon(p) - \overline{\varepsilon}(p)$, where $\overline{\varepsilon}(p)$ is the smooth model of the bending angle corresponding to $\overline{\Psi}'(p)$.

We used $\Delta p = 250$ m and model $\overline{\Psi}'(p)$ was computed as $\Psi'(p)$ smoothed with window Δp. The bending angle error is then estimated as the spectral width (cf. the schematic widths indicated in the two left panels of Fig. 2) (Gorbunov et al. 2005):

$$\delta\varepsilon(p) = \left(\frac{\int|w(p,\xi)|^2\,\xi^2\mathrm{d}\xi}{\int|w(p,\xi)|^2\,\mathrm{d}\xi}\right)^{1/2}. \tag{10}$$

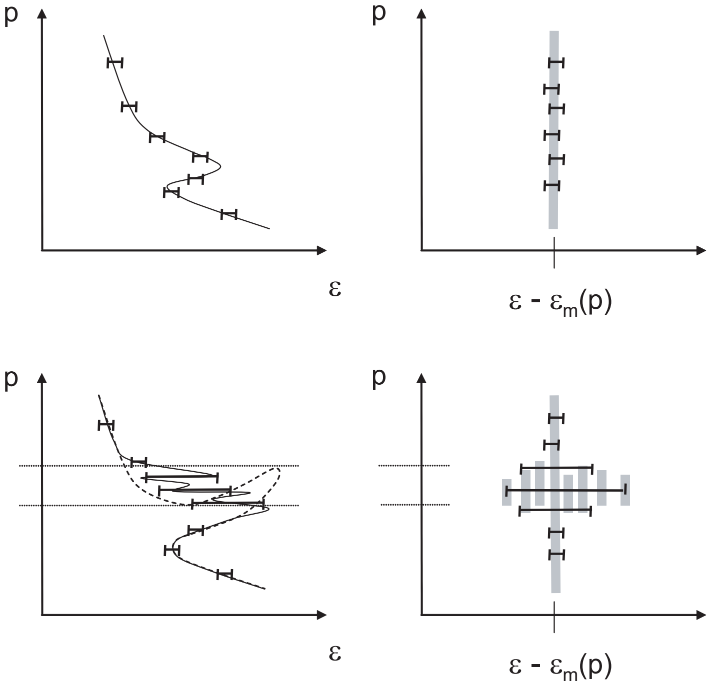

Fig. 2 The principle of the radio holographic (sliding spectrum) estimate of bending angle errors. Top: the situation where the impact parameter is a unique coordinate and where the spectrum is narrow; bottom: the situation where horizontal gradients result in p not being the unique coordinate of the ray manifold. The spectral width allows for the estimate of the bending angle errors.

In non-unique ray regions the sliding spectrum will exhibit a broad structure, which eventually is mapped into a larger bending angle error than in the case where there is only one ray present for a given value of impact parameter.

4 Numerical Simulations

For the numerical simulations we choose an artificial occultation example, where the situation with a non-unique projection of the ray manifold to the impact parameter axis occurred. The simulated occultation was based on an ECMWF re-analysis field from February 5, 1997, UTC 00:00. We simulated soundings of the same region from different azimuths. Azimuths are characterized by the angle between the occultation plane and local north direction. We changed the azimuth from 0° to 330° with a step of 30°. The resulting 12 geometric optical profiles (the simulated "truth") are shown in Fig. 3. These profiles were obtained by the geometric optical ray-tracing. Here bending angles ε are shown as functions of the ray impact heights

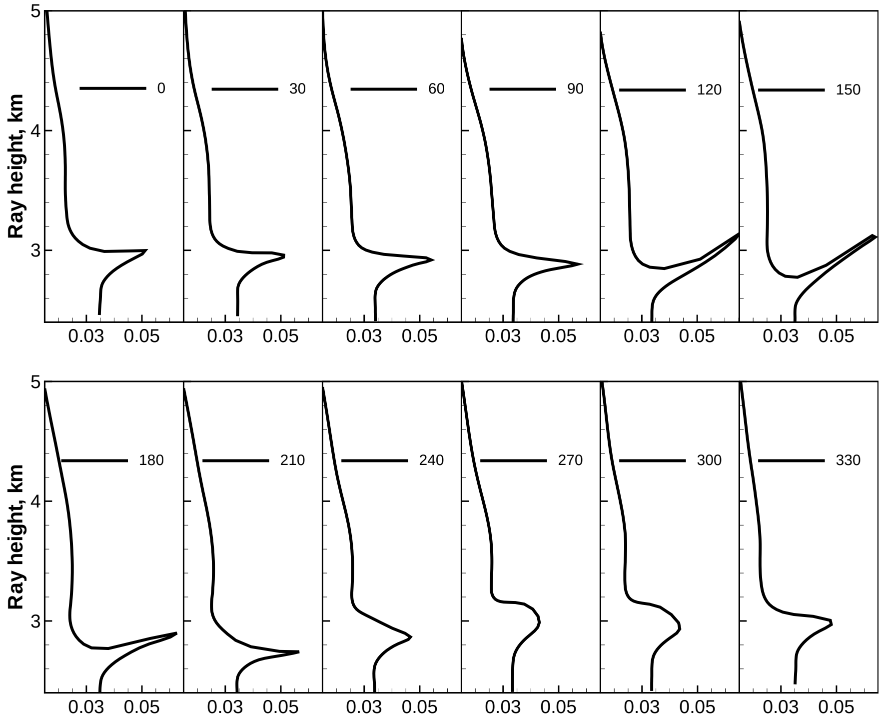

Fig. 3 True simulated bending angle profiles for sounding of the same location 12.4°N, 170.9°W from different azimuths (from 0° to 330°, with a step of 30°). Azimuths are characterized by the angle between the occultation plane and local north direction. The modeling was based on ECMWF fields February 5, 1997, UTC 00:00.

defined as $p - r_{\mathrm{E}}$, where r_{E} is the Earth's curvature radius. The strongest effect of a multi-valued bending angle profile is observed for the azimuths of 120°, 150°, and 180°. A small area of the non-unique ray manifold projection can also be noticed for the azimuth of 0°.

Figure 4 presents the CT-retrieved bending angle profiles. These profiles were retrieved from the artificial RO data simulated by wave optics (Gorbunov 2002). Because the standard CT technique relies upon the impact parameter p being a unique coordinate for the ray manifold, it is incapable of reproducing the true multi-valued bending angle profiles for the azimuths of 120°, 150°, and 180°.

A small area of multi-valued bending angle profile can also be noticed for the azimuth of 0°. Single-valued profiles are retrieved instead. The azimuths, where the assumption of the uniqueness of the ray manifold parameterization with the impact parameter p is broken, are marked by the increased level of the estimated errors based on Eq. (10). In Fig. 5 we show the radio holographic running spectra in the p-domain. It is observed that for the three cases, the spectral width increases in areas of the multi-valued projection of the ray manifold, in accordance with the theoretical

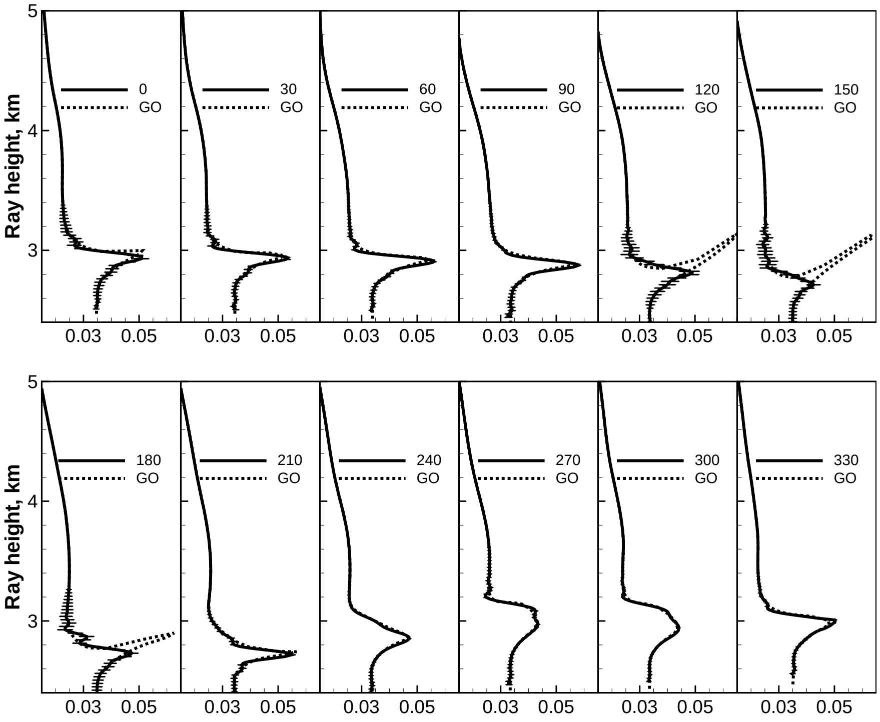

Fig. 4 True simulated bending angle profiles (GO, dashed line), retrieved simulated bending angle profiles (solid lines) and their error estimates for sounding at the same location 12.4°N, 170.9°W from different azimuths (from 0° to 330°, with a step of 30°). Azimuths are characterized by the angle between the occultation plane and local north direction. The modeling was based on ECMWF fields February 5, 1997, UTC 00:00.

expectations. The error bars are, however, smaller than the difference between the true multi-valued and the CT-retrieved bending angle. This is because the spike of the bending angle contains a small portion of energy, which is distributed over a stretched fragment of the ray manifold.

5 Conclusions

The CT/FSI methods use the fact that the impact parameter p is the unique coordinate in the ray space. This condition is universally valid for a spherically symmetrical atmosphere. In an atmosphere with horizontal gradients of refractivity this is not always the case. Strong horizontal gradients result in the variation of the impact parameters along rays rather than being invariant. This is the reason why it is generally impossible to introduce the concept of impact parameter for the atmosphere

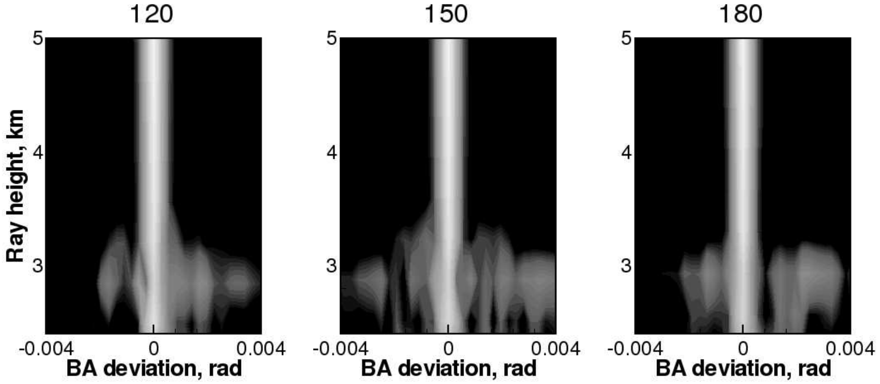

Fig. 5 Sliding spectra in the p-domain for three occultation plane directions, where the bending angle is a multi-valued function of the impact parameter (the text above the figures corresponds to the azimuth value). One observes the broadening of the spectra in the multi-valued multipath regions around 2.75 km to 3.25 km (cf. Fig. 3).

with horizontal gradients. Instead, we introduce the so-called effective impact parameter using the same definition from the Doppler frequency as for the unique impact parameter for a spherically symmetric atmosphere. In the presence of strong horizontal gradients varying with height, perturbations of the effective impact parameter may result in the non-unique projection of the ray manifold to the impact parameter axis. In this case the bending angle can be a multi-valued function of the effective impact parameter. Because the CT/FSI methods rely upon the impact parameter being the unique coordinate along the ray manifold, it follows that in this situation this method will be incapable of correctly retrieving the bending angle profile.

In the general CT formalism it may be possible to introduce a new coordinate, other than impact parameter, such that the ray manifold should have a unique projection to the corresponding axis. Still, this definition of the new coordinate depends on the specific atmospheric fields and, therefore, it cannot be specified a priori. Another opportunity is the estimation of the errors of the retrieved bending angles by means of the radio holographic (sliding spectrum) analysis in the p-domain. Numerical simulations with realistic atmospheric fields from re-analyses of ECMWF show that the error estimate increases in the presence of strong horizontal gradients resulting in multi-valued geometric optical bending angle profiles. This makes the standard CT/FSI techniques complemented with radio holographic error estimates a valuable tool for the analysis of RO data in all situations.

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