Asymptotic wave optics methods in inversion and direct modeling of radio occultations: recent achievements

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Summary. We discuss the recent achievements in the application of asymptotic methods based on Fourier Integral Operators (FIOs) for inversion and direct modeling of radio occultations. We show that FIOs can be derived from the first principles: stationary phase principle and energy conservation. We discuss accurate and approximate solutions for the kernel of the FIOs. The approximations can be used for designing very efficient numerical algorithms, where the FIOs are reduced to a composition of multiplying with reference signals and Fourier transforms. Inversion algorithms use FIO that retrieves the geometric optical ray structure of wave fields. Asymptotic methods of forward modeling are based on inverse FIOs that map the geometric optical ray structure to wave fields. Such algorithms are very fast and significantly reduce numerical inaccuracies, which arise in computation of multiple diffractive integrals.

1 Introduction

Fourier Integral Operators (FIOs) are a very effective means of analysis of wave fields measured in radio occultation experiments [*Gorbunov*, 2002a, b; *Gorbunov and Lauritsen*, 2002; *Jensen et al.*, 2002; *Gorbunov*, 2003; *Gorbunov and Kornblueh*, 2003; *Jensen et al.*, 2003, 2004; *Gorbunov et al.*, 2004; *Gorbunov and Lauritsen*, 2004]. These operators generalize the standard construction of geometrical optics (GO) [*Mishchenko et al.*, 1990]. The basis of GO is the stationary phase principle which describes rays. Rays are curves in the phase space, where there is no multipath, because rays interfering at the some coordinate have different directions, or momenta. Multipath in physical space is characterized by the projection type of the ray manifold. The idea of the canonical transform method is to change coordinates in the phase space so as to change the projection type of the ray manifold [*Gorbunov*, 2002a; *Gorbunov and Lauritsen*, 2004]. The new coordinates should also result in the same minimum action principle [*Kleinert*, 1993]. Fourier Integral Operators generalize the concept of geometric optical canonical transform for wave fields. The stationary phase principle and energy conservation allow for establishing the equation for the kernel of the transform.

The physical principles of stationary phase and energy conservation allow for establishing all the known phase functions of the FIO used in Canonical Transform (CT) method based on FIO of the 1st type, Full-Spectrum Inversion (FSI) method and Phase Matching (PM) [*Jensen et al.*, 2004; *Gorbunov and Lauritsen*, 2004]. The reduction of the FIOs to FT can be done by using some coordinate transform or approximation that linearizes the phase function [*Gorbunov and Lauritsen*, 2004]. The FIO associated with the linearized canonical transform assures high numerical efficiency.

By inverting the FIOs discussed above we can write the transform from the impact parameter representation to the representation of the physical coordinate [*Gorbunov*, 2003]. This results in fast asymptotic algorithm of forward modeling. Such algorithms can be very efficient in numerical applications. Sometimes direct modeling requires multiple phase screens. This is the case when diffraction inside the medium is not negligible, e.g. in the presence of turbulence. The last step from the last phase screen to the LEO orbit is performed by the computation of multiple diffractive integrals. This procedure is very inefficient numerically and, besides, it may be a source of computational inaccuracy. A very efficient solution of the problem of propagation of waves in a vacuum from a phase screen to an observation curve is given by the FIO based on the corresponding linearized CT.

2 Basic principles of Fourier Integral Operators

2.1 Basic Waveforms

Fourier Integral Operators (FIOs) are used for constructing asymptotic solutions of wave problems [*Mishchenko et al.*, 1990]. FIO-based asymptotic solutions generalize the standard geometric optics (GO). A FIO maps a geometric optical solution into to an asymptotic solution of a wave problem. For radio occultations it is of primary importance that this construction can be inverted and the GO solution can be extracted from the measured wave field. This allows for the retrieval of bending angle profiles from measurements of wave fields in multipath regions [*Gorbunov*, 2002a, b]. This also increases the resolution beyond the Fresnel zone, which limits the applicability of the standard geometric optical approach. Below we shall discuss the principles of the theory of FIOs using a few basic physical principles (Fermat's principle and energy conservation).

We will discuss a 2D wave problem. This is a typical approximation for the atmosphere, where the vertical scale is significantly less than the vertical scale. This allows for neglecting effects of diffraction due to gradients transversal to

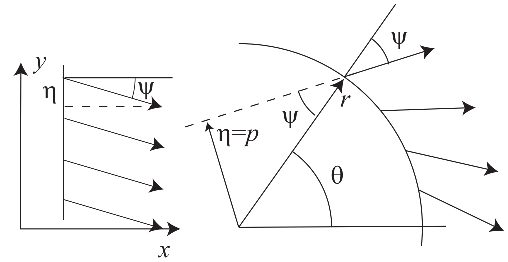

Fig. 1. Basic waveforms: (left) plane waves; (right) cylindrical harmonics.

the occultation plane. We introduce generic coordinates x, y in the plane. Specific choice of the coordinate system can be different. It is only important that axis x is preferred direction of wave propagation. For example, if we discuss a plane incident wave, then it is convenient to use Cartesian coordinates (x, y). For a spherical wave we can choose polar coordinates (r, θ).

Wave field $u(x, y)$ can be expanded with respect to basic wave forms. We begin the consideration from the Cartesian coordinates. Wave field can be represented in the form $u(x, y) = A(x, y) \exp\left(ik\Psi(x, y)\right)$, where $A(x, y)$ is the amplitude, and $\Psi(x, y)$ is the eikonal. This form is convenient for description of the wave field in a single ray are, where the amplitude is a smooth function. For Cartesian coordinates, the most convenient choice of the basic wave forms will be plane waves (Figure 1):

$$u_\eta(x, y) = \tilde{u}(0, \eta) \exp\left[ik\left(\sqrt{1-\eta^2}x + \eta y\right)\right], \tag{1}$$

where $k = \dfrac{2\pi}{\lambda}$ is the wavenumber, $\left(\sqrt{1-\eta^2}, \eta\right)$ is the unity ray direction vector, and $\tilde{u}(0, \eta)$ equals the Fourier transform of the wave field $u(0, y)$ in some source plane, $x = 0$, with respect to y:

$$\tilde{u}(0, \eta) = \sqrt{\frac{-ik}{2\pi}} \int u(0, y) \exp\left(-ik\eta y\right) dy. \tag{2}$$

The wave vector of a plane wave equals $\mathbf{k} = (k_x, k_y) = \left(k\sqrt{1-\eta^2}, k\eta\right)$. Momentum equals $\eta = \sin\psi$. The element of optical path along the ray equals simply the element of the length (Figure 1):

$$dx = \sqrt{1-\eta^2}ds, \quad dy = \eta\, ds, \tag{3}$$

$$d\Psi = \eta\, dy - H\, dx = \sqrt{1-\eta^2}\, dx + \eta\, dy = ds. \tag{4}$$

Each plane wave corresponds to a family of parallel rays.

A similar consideration can be performed for polar coordinates r, θ. In this case the basic wave forms are cylindrical waves (Figure 1):

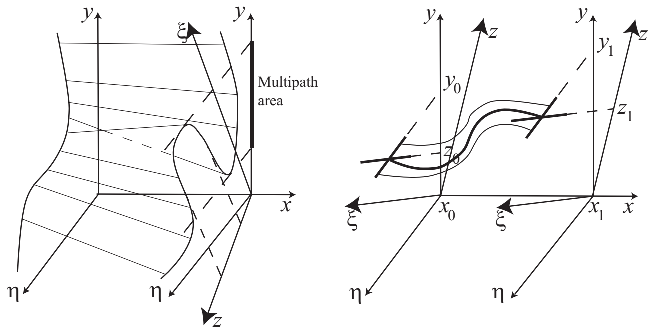

Fig. 2. (left) Schematic ray manifold in the phase space. The ray manifold evolves along the coordinate x. The type of its projection can be changed by choosing new coordinates (z, ξ) in the phase space. (right) Virtual variations of ray paths in different canonical coordinates in the phase space.

$$u_\eta(r, \theta) = \tilde{u}(r_0, \eta)\sqrt{\frac{r_0}{r}} \exp\left[ik\left(\sqrt{1 - \frac{\eta^2}{r^2}}r + \eta\,\theta\right)\right], \tag{5}$$

and the momentum equal ray impact parameter, $\eta = r \sin\psi = p$.

If we consider an arbitrary wave field $u(x, y) = A(x, y) \exp\left(ik\Psi(x, y)\right)$, then we can define ray direction at every point as $\eta(y) = \hat{\eta}u(x, y) \approx \dfrac{\partial \Psi}{\partial y}$. This definition only works if there is a single ray, and the amplitude is a smooth function. In this case the derivative of the amplitude can be neglected. Function $\eta(y)$ defines the ray manifold in the phase space with coordinates (y, η). The single-ray propagation corresponds to the situation when the ray manifold has an unique projection to the axis y. Multipath propagation means that $\eta(y)$ is a multi-valued function, and it cannot be expressed as $\dfrac{\partial \Psi}{\partial y}$.

From the geometric optical view point, the problem of multipath can be solved by another choice of coordinate and momentum (y, η) in the phase space. We need to parameterize the phase space by a different coordinate and momentum (z, ξ) in such a way that the projection of the ray manifold to the new axis z is unique, and therefore, $\xi(z)$ is a single-valued function (Figure 2). Generally speaking, when introducing the new coordinates (z, ξ) we have to define the new Hamilton function. We also need to find the corresponding transform $\hat{\Phi}$ of the wave field. The space of possible transforms is defined by two fundamental principles; 1) Fermat principle, 2) energy conservation.

2.2 Stationary Phase Principle and Canonical Transforms

The Fermat principle should hold both in the old and new coordinates [*Arnold*, 1978; *Kleinert*, 1993]:

$$\delta\Psi = \delta \int\limits_{(x_0,y_0)}^{(x_1,y_1)} d\Psi = \delta \int\limits_{(x_0,y_0)}^{(x_1,y_1)} \eta\, dy - H\, dx = 0, \tag{6}$$

$$\delta\Psi' = \delta \int\limits_{(x_0,z_0)}^{(x_1,z_1)} d\Psi' = \delta \int\limits_{(x_0,z_0)}^{(x_1,z_1)} \xi\, dz - H'\, dx = 0, \tag{7}$$

where the integrals are taken along the same physical ray parameterized by different coordinates. The ray connects points (y_0, η_0) and (y_1, η_1), or (z_0, ξ_0) and (z_1, ξ_1). The variations of ray paths have the following restrictions: $\delta y_{0,1} = 0$ and $\delta z_{0,1} = 0$ (Figure 2). The variations of momenta are not restricted (we allow arbitrary variations of the ray direction).

Because the new coordinate z is a function of (y, η), the boundary condition $\delta z = 0$ may imply some variation $\delta y \neq 0$, and vice versa. To establish the relation between $d\Psi$ and $d\Psi'$ we consider arbitrary variations of the optical path form $\delta\Psi$ and $\delta\Psi'$ in the vicinity of a stationary path, not restricted with conditions $\delta y_{0,1} = 0$ and $\delta z_{0,1} = 0$:

$$\delta\Psi = \eta\, \delta y|_{x_0}^{x_1}, \quad \delta\Psi' = \xi\, \delta z|_{x_0}^{x_1}. \tag{8}$$

We require that $d\Psi' - d\Psi = \xi\, dz - \eta\, dy - (H' - H)\, dx$ should be equal to a full differential dS [*Arnold*, 1978; *Kleinert*, 1993]. For arbitrary variations of an arbitrary trajectory in the phase space we have then the following relation:

$$\delta\Psi' - \delta\Psi = \delta \int\limits_{x_0}^{x_1} dS = \delta S(y, z)|_{x_0}^{x_1} = \xi\, \delta z - \eta\, \delta y|_{x_0}^{x_1}. \tag{9}$$

Therefore, if for some path $\delta\Psi = 0$ with the boundary condition $\delta y_{0,1} = 0$, then for the same path $\delta\Psi' = 0$ with boundary condition $\delta z_{0,1} = 0$. If we consider a cross section of the phase space frozen at some fixed x, then we can write the reduced equation:

$$dS = \xi\, dz - \eta\, dy, \tag{10}$$

$$\frac{\partial S}{\partial z} = \xi, \quad \frac{\partial S}{\partial y} = -\eta \tag{11}$$

A transform from (y, η) to (z, ξ) such that $\xi\, dz - \eta\, dy$ equals a full differential dS is termed *canonical*, $S(z, y)$ being its *generating function* [*Arnold*, 1978].

2.3 Fourier Integral Operator of 2nd Type

Consider now a complex integral transform on the wave field $u(y)$:

$$v(z) = \hat{\Phi}_2 u(z) = \sqrt{\frac{-ik}{2\pi}} \int a_2(z, y) \exp\left(ikS_2(z, y)\right) u(y) dy. \tag{12}$$

We will refer to this operator as to a FIO of the 2nd type [*Gorbunov and Lauritsen*, 2002, 2004]. The transformed wave field $v(z)$ can be written in the form $A'(z)\exp(ik\Psi'(z))$. Our aim is to find a transform of the wave field that implements a canonical transform, i.e. given the momentum $\dfrac{\partial\Psi(y)}{\partial y} = \eta$ of the wave field $u(y)$, the momentum $\dfrac{\partial\Psi'(z)}{\partial z}$ of the transformed wave field $v(z)$ should be equal to ξ.

2.4 Energy conservation

Another important requirement is that this transform conserves the energy of the wave field [*Egorov*, 1985; *Egorov et al.*, 1999]:

$$\int v\, v^* dz = \int u\, u^* dy. \tag{13}$$

From this it follows that the conjugated operator $\hat{\Phi}_2^*$ should be equal to the inverse operator $\hat{\Phi}_2^{-1}$, because by definition

$$\int \hat{\Phi}_2 u \left(\hat{\Phi}_2 u\right)^* dz = \int u \left(\hat{\Phi}_2^* \hat{\Phi}_2 u\right)^* dz, \tag{14}$$

$$\hat{\Phi}_2^* v(y) = \hat{\Phi}_2^{-1} v(z) = \sqrt{\frac{ik}{2\pi}} \int a(z,y)\exp\left(-ikS_2(z,y)\right) v(z) dz. \tag{15}$$

Substituting $u(y) = \delta(y - y_0)$ and considering $\hat{\Phi}u(z)$ we have $\Psi'(z) = S_2(z, y_0)$. From here it follows that $\dfrac{\partial S_2}{\partial z} = \xi$. Substituting $v(z) = \delta(z - z_0)$ and considering $\hat{\Phi}^{-1}v(z)$ we have $\Psi(y) = -S_2(z_0, y)$. From here it follows that $\dfrac{\partial S_2}{\partial y} = -\eta$. Thus we see that $S_2(z, y)$ equals the generating function of the canonical transform $S(z, y)$.

The simplest form of a Fourier Integral Operator is the Fourier transform:

$$S_2(z,y) = -zy, \tag{16}$$

$$\xi = \frac{\partial S_2}{\partial z} = -y; \quad -\eta = \frac{\partial S_2}{\partial y} = -z, \tag{17}$$

which implements a $\pi/2$ rotation of the phase plane $(y, \eta) \to (z = \eta, \xi = -y)$.

2.5 Fourier Integral Operator of 1st Type

Another, 1st, type of FIO can be expressed as a composition of two FIOs of the 2nd type, one of which is the Fourier transform [*Egorov*, 1985]:

$$v(z) = \hat{\Phi}_1 u(z) = \sqrt{\frac{ik}{2\pi}} \int a_1(z,\eta)\exp\left(ikS_1(z,\eta)\right) \tilde{u}(\eta) d\eta, \tag{18}$$

where the equation for the generating function S_2 is similar to that for S_1:

$$dS_1 = \xi\, dz + y\, d\eta, \tag{19}$$

$$\frac{\partial S_1}{\partial z} = \xi, \quad \frac{\partial S_1}{\partial \eta} = y. \tag{20}$$

3 Processing Radio Occultations

3.1 Phase Function: Accurate and Approximate Solutions

The application of the technique of FIOs for processing radio occultation data uses the fact that impact parameters uniquely characterize rays in a spherically symmetric atmosphere [*Gorbunov*, 2002a, b; *Gorbunov and Lauritsen*, 2002; *Jensen et al.*, 2002; *Gorbunov*, 2003; *Gorbunov and Kornblueh*, 2003; *Jensen et al.*, 2003, 2004; *Gorbunov et al.*, 2004; *Gorbunov and Lauritsen*, 2004]. Equations (11,20) can be directly applied for the derivation of the phase functions [*Gorbunov and Lauritsen*, 2004]:

$$S_2(p, y) = -\int \eta(p, y) dy, \quad S_1(p, \eta) = \int y(p, \eta) d\eta. \tag{21}$$

Now consider the expression for the derivative of the optical path of a radio occultation signal:

$$\dot{\Psi} = \eta(p, t) \equiv \dot{\theta} p + \frac{\dot{r}_L}{r_L}\sqrt{r_L^2 - p^2} + \frac{\dot{r}_G}{r_G}\sqrt{r_G^2 - p^2}. \tag{22}$$

This allows for the derivation of the exact phase function [*Jensen et al.*, 2004]:

$$\begin{aligned} S_2(p, t) = &-\int \left(p d\theta + \frac{dr_G}{r_G}\sqrt{r_G^2 - p^2} + \frac{dr_L}{r_L}\sqrt{r_L^2 - p^2} \right) = \\ &= -p\theta - \sqrt{r_G^2 - p^2} + p \arccos\frac{p}{r_G} - \sqrt{r_L^2 - p^2} + p \arccos\frac{p}{r_L}. \end{aligned} \tag{23}$$

The corresponding momentum equals minus refraction angle:

$$\xi(p) = \frac{\partial \Psi'(p)}{\partial p} = \frac{\partial S_2(p, t_s(p))}{\partial p} = -\theta + \arccos\frac{p}{r_G} + \arccos\frac{p}{r_L} = -\epsilon(p). \tag{24}$$

Precise phase function is inconvenient for numerical implementation, because the corresponding operator cannot be reduced to the Fourier transform. To reduce this operator to the Fourier transform, it is necessary to use some approximation. The simplest approximation is that implemented in the Full-Spectrum Inversion method [*Jensen et al.*, 2003]:

$$S_2(p, \theta) = -p\theta - \int \left(\frac{\dot{r}_G}{r_G}\sqrt{r_G^2 - p_m^2} + \frac{\dot{r}_L}{r_L}\sqrt{r_L^2 - p_m^2} \right) d\theta \equiv -p\theta + \int F(\theta) d\theta, \tag{25}$$

where $p_m = p_m(\theta)$ is the a priori model of impact parameter variation. The corresponding momentum $\xi(p)$ equals minus satellite-to-satellite angle, $\partial S_2/\partial p = -\theta$.

More accurate approximation is based on the linearization of the canonical transform from (t, η) to (p, ξ) in the vicinity of some model $[p_0(t),\ \eta_0(t)]$ [*Gorbunov and Lauritsen*, 2004]:

$$\tilde{p}(t,\eta) = p_0 + \frac{\partial p_0}{\partial \eta}(\eta - \eta_0) = f(t) + \frac{\partial p_0}{\partial \eta}\eta, \tag{26}$$

$$f(t) = p_0 - \left(\dot{\theta} - \frac{\dot{r}_G}{r_G}\frac{p_0}{\sqrt{r_G^2 - p_0^2}} - \frac{\dot{r}_L}{r_L}\frac{p_0}{\sqrt{r_L^2 - p_0^2}}\right)^{-1}\eta_0. \tag{27}$$

Instead of (t, η) we introduce scaled coordinate and frequency (Υ, σ):

$$d\Upsilon = \left(\frac{\partial p_0}{\partial \eta}\right)^{-1} dt = d\theta - \frac{dr_G}{r_G}\frac{p_0}{\sqrt{r_G^2 - p_0^2}} - \frac{dr_L}{r_L}\frac{p_0}{\sqrt{r_L^2 - p_0^2}}, \tag{28}$$

$$\sigma = \frac{\partial p_0}{\partial \eta}\eta. \tag{29}$$

This scaling allows for the derivation of the linear canonical transform and its generating function:

$$\tilde{p} = f(\Upsilon) + \sigma, \quad \xi = -\Upsilon, \tag{30}$$

$$S_2(\tilde{p}, \Upsilon) = -\int \sigma(\tilde{p}, \Upsilon) d\Upsilon = -\tilde{p}\Upsilon + \int f(\Upsilon)\, d\Upsilon. \tag{31}$$

The FIO with this phase function is very similar to FSI, but it uses more precise approximation, and the definition of scaled coordinate Υ instead of θ takes into account the deviation of the trajectory from circle.

Figure 3 shows the profiles of temperature, humidity, real refractivity and specific absorption for GPS frequencies from a high-resolution radio sonde data. This profile was used for modeling a spherically symmetrical atmosphere. A radio occultation experiment was simulated using multiple phase screen technique, and the simulated data were processed by CT method based on the FIO with the phase function (31). The simulation was performed for GPS frequencies. The results of processing the simulated data are shown in Figure 3. The complicated profile of bending angle is retrieved with a good accuracy.

4 Forward Modeling

4.1 Wave Propagation in Atmosphere

Fourier integral operators can also be used for asymptotic direct modeling [*Gorbunov*, 2003; *Gorbunov and Lauritsen*, 2004]. The FIOs $\hat{\Phi}_{1,2}$ can be easily inverted: $\hat{\Phi}_{1,2}^{-1} = \hat{\Phi}_{1,2}^{*}$. If we use the representation of approximate impact

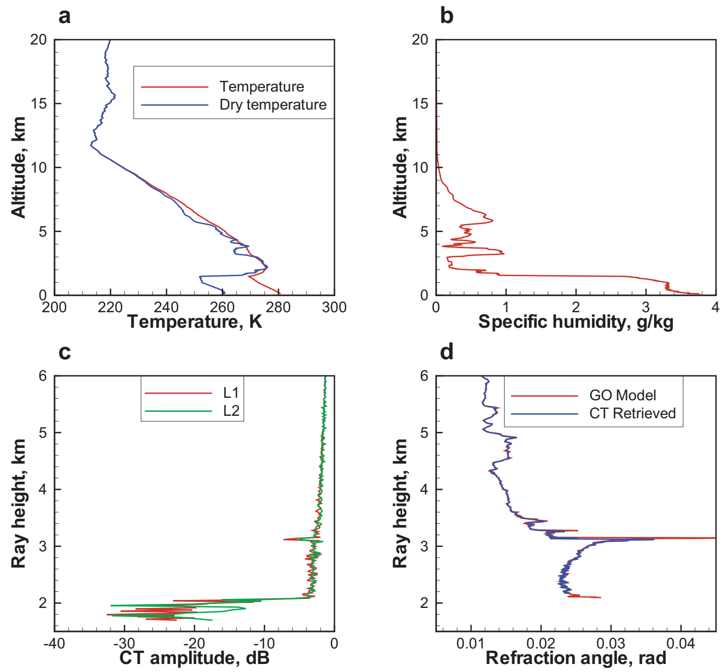

Fig. 3. Simulated occultation event for a high resolution radio sonde data: (a) temperature, T, and dry temperature, T_{dry}, (b) specific humidity, q, (c) CT amplitudes for the two channels, and (d) refraction angles, comuted by the GO model and retrieved by the CT method.

parameter $\tilde{p}$, then the direct model is especially efficient. Given a 3D atmospheric model, we first perform geometric optical modeling, and iteratively find the trajectory point $\Upsilon_s(\tilde{p})$, where the ray with the impact parameter $p(\tilde{p})$ is observed. The wave function in the $\tilde{p}$-representation is then equal to $w(\tilde{p}) = A'(\tilde{p}) \exp\left(-ik \int \Upsilon_s(\tilde{p}) d\tilde{p}\right)$, where the amplitude $A'(\tilde{p})$ equals a normalizing constant in the light zone and 0 in the geometric optical shadow. This function is then mapped into the Υ-representation by the inverse FIO [*Gorbunov and Lauritsen*, 2004]:

$$u(\Upsilon) = \sqrt{\frac{ik}{2\pi}} \exp\left(-ik \int\limits_0^{\Upsilon} f(\Upsilon')\, d\Upsilon'\right) \int \exp(ik\tilde{p}\Upsilon)\, a_2(\Upsilon_s(\tilde{p}), \tilde{p})\, w(\tilde{p})\, d\tilde{p}, \tag{32}$$

For modeling atmospheric absorption, the amplitude $A'(\tilde{p})$ must also be multiplied by a factor of $\exp\left(-k \int n'' ds\right)$, where n'' is the imaginary part of refractive index, and the integral is taken along the ray with the impact parameter $p(\tilde{p})$. A similar direct modeling algorithm can be constructed by inverting the operator used in the FSI method.

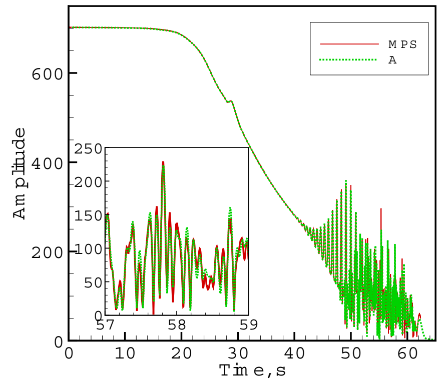

Fig. 4. Validation of asymptotic direct modeling. Amplitude of simulated radio occultation signal as function of time: (1) MPS simulation (solid line) and (2) asymptotic simulation based on the FIO2 (A, dotted line).

For the validation of the asymptotic direct modeling we performed numerical simulations with a simple spherically-symmetrical phantom (refractive index field model). The phantom represents an exponential model with a quasi-periodical perturbation:

$$n(z) = 1 + N_0 \exp\left(-\frac{z}{H}\right)\left[1 + \alpha \cos\left(\frac{2\pi z}{h}\right)\exp\left(-\frac{z^2}{L^2}\right)\right], \qquad (33)$$

where z is the height above the Earth's surface, $N_0 = 300 \times 10^{-6}$ is the characteristic refractivity at the Earth's surface (300 N-units), $H = 7.5$ km is the characteristic vertical scale of refractivity field, $\alpha = 0.003$ is the relative magnitude of the perturbation, $h = 0.3$ km is the period of the perturbation, $L = 3.0$ km is the characteristic height of the perturbation area. This phantom was smoothly combined with the MSIS climatological model above 20 km. We simulated radio occultation signals using MPS and the asymptotic solution for the frequency 9.7 GHz, which is intended to be used in LEO-LEO occultations. The results of the comparison of the amplitude of the simulated wave field for these two modeling techniques are presented in Figure 4. The peculiarity of the amplitude around 28.5 s is due to the transfer from MSIS to the test phantom. Between 40 and 47.5 s the amplitude indicates large-scale oscillations reproducing the oscillations of the refractivity profile. In this area there is no multipath propagation. After 47.5 s we notice increasing small-scale scintillations due to emergence of multipath propagation. The occultation fragment from 57 to 59 s with strong multipath scintillations is enlarged and shown separately. Figure 4 illustrates a good agreement of both these simulation techniques.

4.2 Wave Propagation in Vacuum

Asymptotic forward modeling has the following applicability limitation: it cannot be used for modeling effects of diffraction inside the atmosphere. Accurate account of diffraction on small-scale atmospheric structures is necessary e.g. for modeling wave propagation in a turbulent atmosphere. In this case it is necessary to apply multiple phase screen technique. Previously, the step from the last phase screen to the LEO orbit is performed by the computation of multiple diffractive integrals. The computation of diffractive integrals is not only very ineffective numerically, it is also a source of computational inaccuracies. However, using the technique of FIOs it is possible to construct an asymptotic solution of wave propagation in a vacuum from a straight phase screen to an arbitrary observation curve that can be reduced to the Fourier transform.

Consider the wave field $u_0(y)$ in the phase screen plane, where y is the vertical coordinate, and the observation curve $X(t), Y(t)$ (Figure 5). The accurate solution based on the plane wave expansion of the source field can easily written:

$$u(t) = \sqrt{\frac{ik}{2\pi}} \int \exp\left(ikX(t)\sqrt{1-\eta^2} + ikY(t)\eta\right) \tilde{u}_0(\eta) d\eta. \tag{34}$$

This is FIO of the first type with the following phase function:

$$S_1(t,\eta) = X(t)\sqrt{1-\eta^2} + Y(t)\eta. \tag{35}$$

This operator transforms the wave field from the representation (y,η) in the source plane to the representation (t,σ) on the observation curve. We can write the standard differential equation for the phase function:

$$dS_1 = \sigma dt + y d\eta, \tag{36}$$

$$\frac{\partial^2 S_1}{\partial t\,\partial\eta} = \frac{\partial\sigma}{\partial\eta} = \frac{\partial y}{\partial t} \tag{37}$$

which corresponds to the following equations describing straight rays:

$$\sigma = \frac{\partial S}{\partial t} = \dot{X}(t)\sqrt{1-\eta^2} + \dot{Y}(t)\eta, \tag{38}$$

$$y = \frac{\partial S}{\partial \eta} = Y(t) - X(t)\frac{\eta}{\sqrt{1-\eta^2}}. \tag{39}$$

Operator (34) provides an accurate solution. However, it cannot be reduced to the Fourier transform in general case. For a vertical observation trajectory, $X(t) = const$, we can parameterize the observation trajectory with coordinate Y, and the operator will turn into a Fourier transform.

For a general case, we will construct an approximation based on the linearization of the canonical transform $(y,\eta) \to (t,\sigma)$. For this we introduce smooth model of the ray structure $[t_0(\eta), \sigma_0(\eta), y_0(\eta), \eta_0(t)]$. Then the canonical transform can be linearized:

$$t = t_0 + \frac{\partial t_0}{\partial y}(y - y_0) = t_0 - \frac{\partial t_0}{\partial y} y_0 + \frac{\partial t_0}{\partial y} y = f(\eta) + z, \tag{40}$$

$$\sigma = \sigma_0 + \frac{\partial \sigma_0}{\partial \eta}(\eta - \eta_0) = \sigma_0 - \xi_0 + \xi = g(t) + \xi \tag{41}$$

Instead of (y, η), we introduced scaled coordinate and momentum using (37):

$$z = \frac{\partial t_0}{\partial y} y \quad d\xi = \left(\frac{\partial t_0}{\partial y}\right)^{-1} d\eta \tag{42}$$

This allows for the derivation of the phase function:

$$dS = \sigma dt + z d\xi = (g(t) + \xi)dt + (t - f(\xi))d\xi \tag{43}$$

$$S(t, \xi) = \int g(t)dt - \int f(\xi)d\xi + t\xi \tag{44}$$

The approximate FIO is represented as a composition of multiplication with the first reference signal $a(t_0(\xi), \xi) \exp\left(-ik \int f(\xi)d\xi\right)$, Fourier transform, and multiplication with the second reference signal $\exp\left(ik \int g(t)dt\right)$:

$$u(t) = \sqrt{\frac{ik}{2\pi}} \exp\left(ik \int g(t)dt\right) \times \\ \times \int a(t_0(\xi), \xi) \exp(ikt\xi) \exp\left(-ik \int f(\xi)d\xi\right) \tilde{u}_0(\eta(\xi))d\xi \tag{45}$$

We used the same radio sonde profile as above for validation of the algorithm of wave propagation from a phase screen to LEO orbit based on FIO (45). Figure 5 presents a comparison of the amplitude of the wave field computed by the FIO with that computed by the standard algorithm based on Fresnel integrals. The results of the two methods are in a very good agreement. However, the occultation data obtained by Fresnel integral technique break at 62 s, while the data obtained by FIO technique still continue. This is linked with the difficulty of the identification of the stationary point of the Fresnel integral near the border of shadow zone, where the signal becomes weak. In this example, we observe a sharp spike of refraction angle. Such a spike produces a very weak signal, which may be difficult to compute by Fresnel integral technique. However, such signals are not a problem for the FIO technique.

5 Conclusions

FIOs provide a generalization of geometric optical canonical formalism for wave optics. This allows for using them for solving problem of disentangling

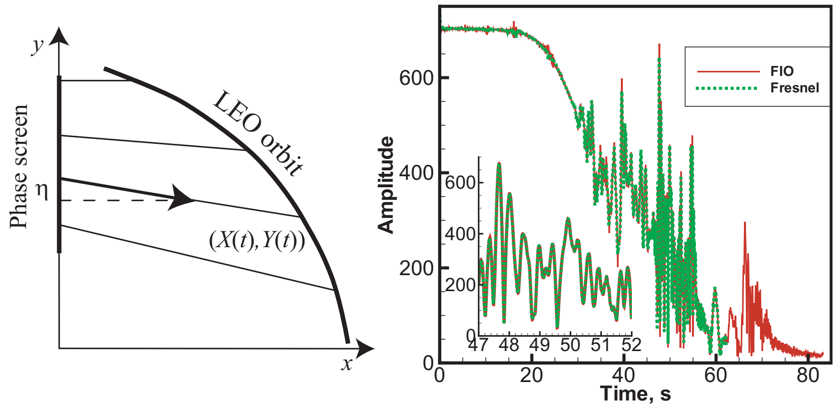

Fig. 5. (left) Geometry of wave propagation from phase screen to LEO orbit. (right) Validation of FIO-based algorithm of wave propagation from phase screen to LEO orbit. Amplitude of simulated radio occultation signal as function of time: (1) FIO algorithm (solid line) and (2) Fresnel integrals (dotted line).

multipath by finding an unique projection of ray manifold. Another, equivalent view of FIOs is based on signal processing approach and frequency matching principle: at the stationary point of the oscillating integral the frequency of the signal is matched by the frequency of the oscillating kernel. This principle allows for sorting signal components with different instantaneous frequencies.

The practical meaning of FIOs for inversion and forward modeling of radio occultation data cannot be underestimated. 1) FIOs provide high accuracy and vertical resolution in the retrieval of refraction angles. 2) FIOs allow for the retrieval of transmission due to atmospheric absorption. 3) Using reasonable approximations, it is possible to reduce the FIOs to a composition of multiplication with a reference signal and Fourier transforms. This is important for achieving high numerical efficiency of inversion algorithms. 4) FIOs can be very effectively used in the forward modeling of radio occultation signals.

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References

Arnold, V. I., *Mathematical Methods of Classical Mechanics*, Springer-Verlag, New York, 1978.

Egorov, Y. V., *Lectures on Partial Differential Equations. Additional Chapters*, Moscow State University Press, Moscow, 1985, (In Russian).

Egorov, Y. V., A. I. Komech, and M. A. Shubin, *Elements of the Modern Theory of Partial Differential Equations*, Springer-Verlag, Berlin, 1999.

Gorbunov, M. E., Canonical transform method for processing GPS radio occultation data in lower troposphere, *Radio Sci.*, *37*, 9–1–9–10, doi:10.1029/2000RS002,592, 2002a.

Gorbunov, M. E., Radio-holographic analysis of Microlab-1 radio occultation data in the lower troposphere, *J. Geophys. Res. - Atm.*, *107*, 7–1–7–10, doi: 10.1029/2001JD000,889, 2002b.

Gorbunov, M. E., An asymptotic method of modeling radio occultations, *J. Atm. Sol.-Terrestr. Phys.*, *65*, 1361–1367, 2003.

Gorbunov, M. E., and L. Kornblueh, Analysis and validation of Challenging Minisatellite Payload (CHAMP) radio occultation data, *J. Geophys. Res.*, *108*, 4584, doi:10.1029/2002JD003,175, 2003.

Gorbunov, M. E., and K. B. Lauritsen, Canonical transform methods for radio occultation data, *Scientific Report 02-10*, Danish Meteorological Institute, Copenhagen, Denmark, 2002, http://www.dmi.dk/dmi/Sr02-10.pdf.

Gorbunov, M. E., and K. B. Lauritsen, Analysis of wave fields by Fourier Integral Operators and its application for radio occultations, *Radio Sci.*, *39*, RS4010, doi:10.1029/2003RS002,971, 2004, accepted.

Gorbunov, M. E., H.-H. Benzon, A. S. Jensen, M. S. Lohmann, and A. S. Nielsen, Comparative analysis of radio occultation processing approaches based on Fourier integral operators, *Radio Sci.*, 2004, accepted.

Jensen, A. S., H.-H. Benzon, and M. S. Lohmann, A new high resolution method for processing radio occultation data, *Scientific Report 02-06*, Danish Meteorological Institute, Copenhagen, Denmark, 2002.

Jensen, A. S., M. S. Lohmann, H.-H. Benzon, and A. S. Nielsen, Full spectrum inversion of radio occultation signals, *Radio Sci.*, *38*, 6–1–6–15, doi: 10.1029/2002RS002,763, 2003.

Jensen, A. S., M. S. Lohmann, A. S. Nielsen, and H.-H. Benzon, Geometrical optics phase matching of radio occultation signals, *Radio Sci.*, *39*, RS3009, doi: 10.1029/2003RS002,899, 2004.

Kleinert, H., *Pfadintegrale in Quantenmechanik, Statistik und Polymerphysik*, Wissenschaftsverlag, Manncheim, Leipzig, Wien, Zürich, 1993.

Mishchenko, A. S., V. E. Shatalov, and B. Y. Sternin, *Lagrangian manifolds and the Maslov operator*, Springer-Verlag, Berlin - New York, 1990.