

# Refractivity fluctuations as a systematic error source in radio occultations

M. E. Gorbunov, V. V. Vorob'ev

Institute of Atmospheric Physics, Russian Academy of Sciences,  
Moscow, Russia



K. B. Lauritsen

Danish Meteorological Institute, Copenhagen, Denmark



# Contents

1. Negative N-bias: observations and possible sources
2. Negative N-bias: One more source is related to fluctuations of refractivity as was early recognized by Eshleman and Haugstad (1977)
3. Numerical simulations: phase screens with a fluctuating component
4. Turbulence model
5. Numerical simulations: systematic effects and its dependence on  $C_N^2$
6. Comparison of COSMIC–ECMWF statistics with simulations
7. Conclusions

# Negative N-bias

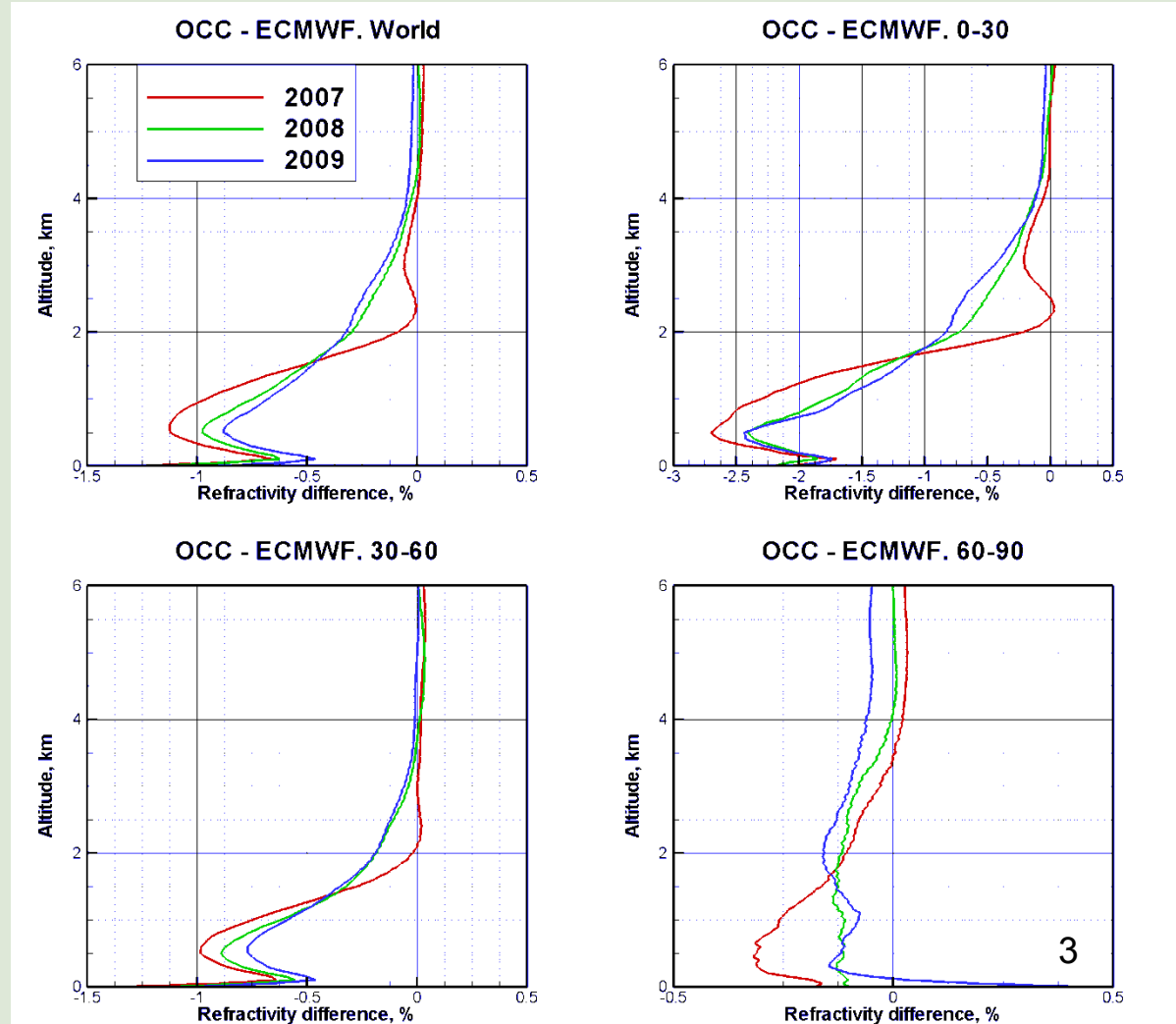
## COSMIC/OCC–ECMWF differences: 2007, 2008, 2009

Change in ECMWF cycle 32r3, 2008:

- i) COSMIC RO assimilated to surface
- ii) updated convection and entrainment physics

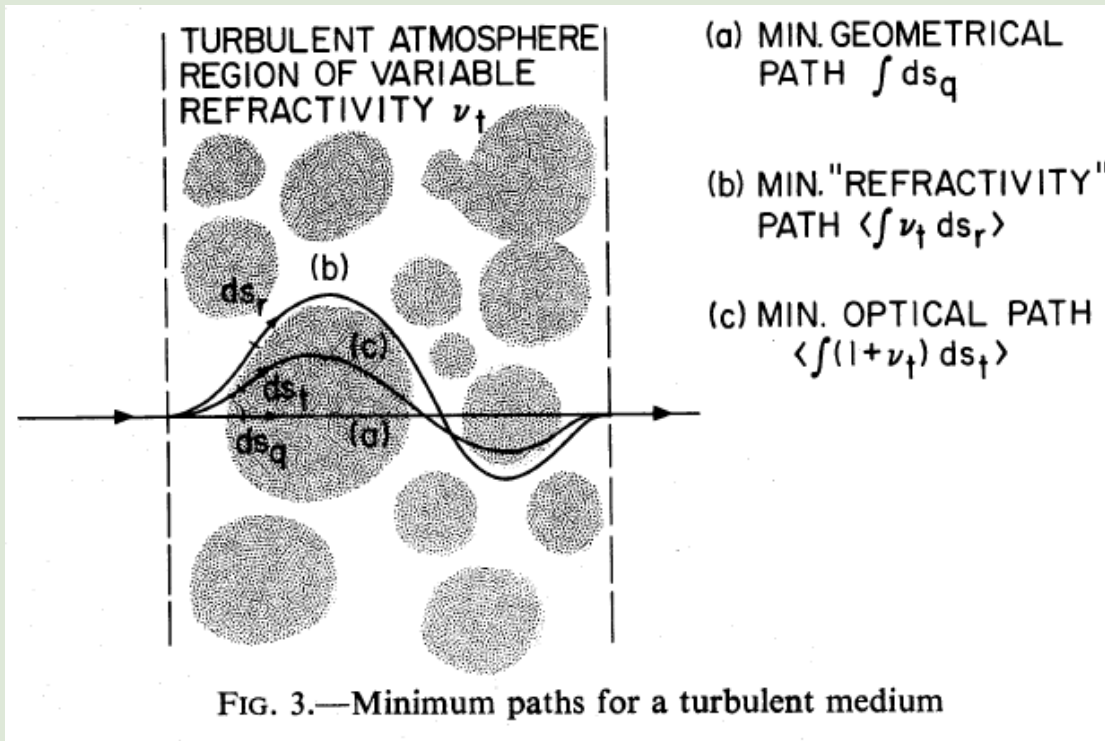
Sources of negative N-bias:

- Super refraction
- Deep occultations
- Horizontal gradients (inversion with Wigner distribution function; TFMs)
- ...



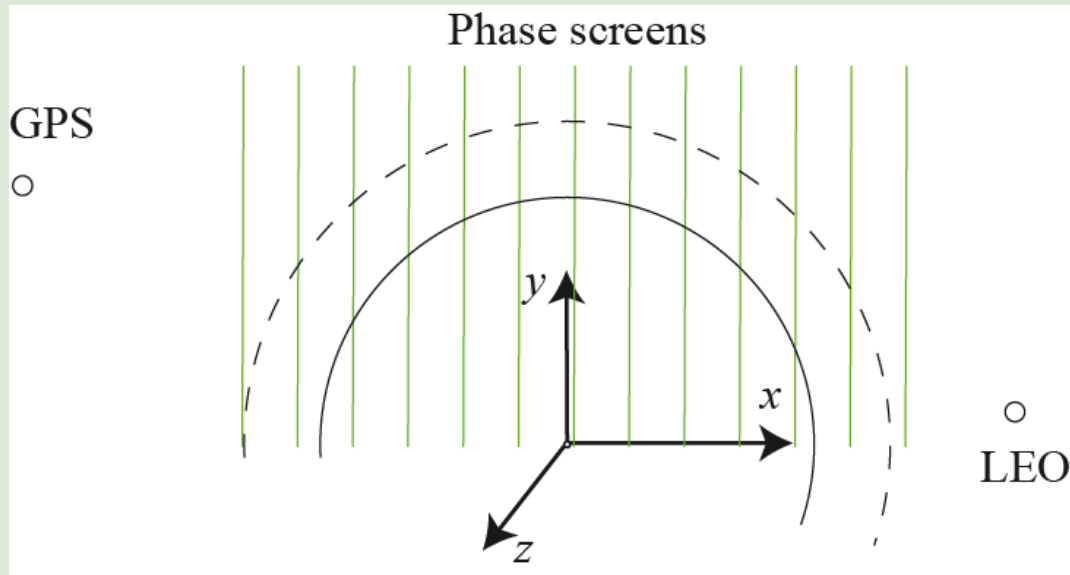
# Negative N-bias

Von R. Eshleman and Bjarne S. Haugstad, *Lowest-order Average Effect of Turbulence on Atmospheric Profiles Derived from Radio Occultation*,  
 The Astrophysical Journal, 214:928-933, 1977



Due to the Fermat principle, each ray tends to choose a path through negative fluctuations of refractivity

# Phase screens



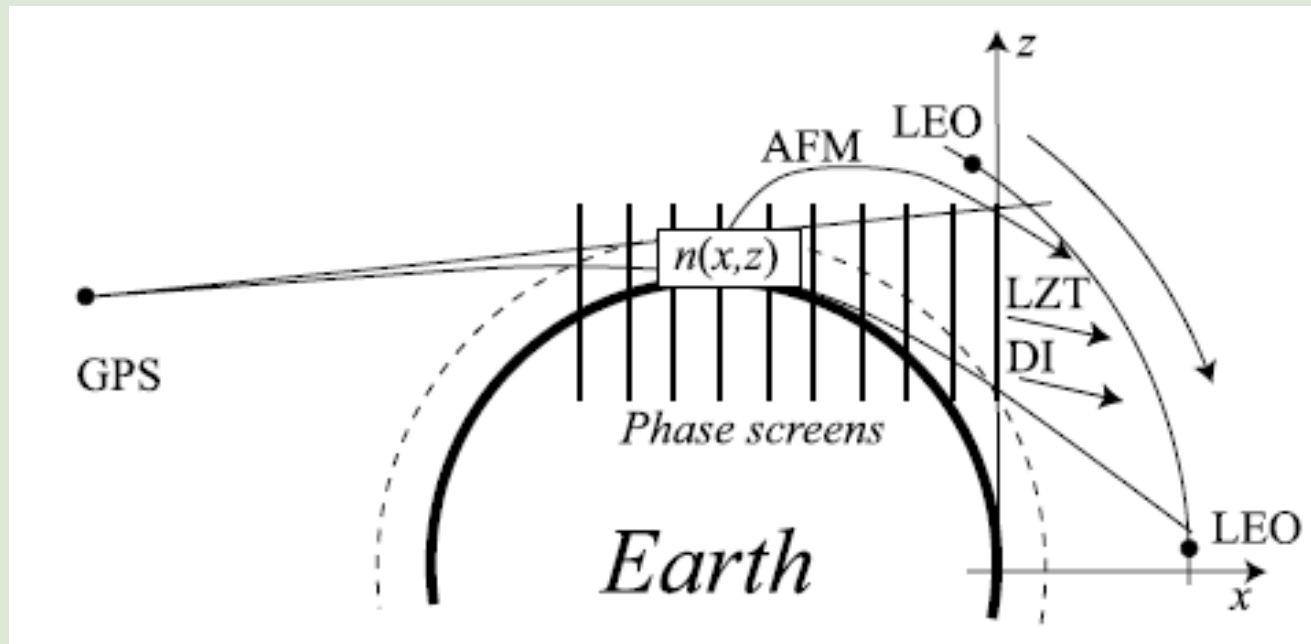
1. Vacuum propagation from GPS to the 1st screen
2. Screen-to-screen propagation:

$$u_{j+1} = \exp \left( ik \int_{x_j}^{x_{j+1}} n(x, y) dx \right) F_{\eta \rightarrow y}^{-1} \exp \left( \sqrt{1 - \eta^2} \Delta x \right) F_{y \rightarrow \eta} u_j$$

3. Vacuum propagation from the last screen to the LEO orbit

# Propagation from the Last Screen

M. E. Gorbunov and K. B. Lauritsen, *Linearized Zverev Transform and its application for modeling radio occultations*, Radio Science, V. 42, RS3023, doi: 10.1029/2006RS003590, 2007



LZT:  
Linear  
Zverev  
transform

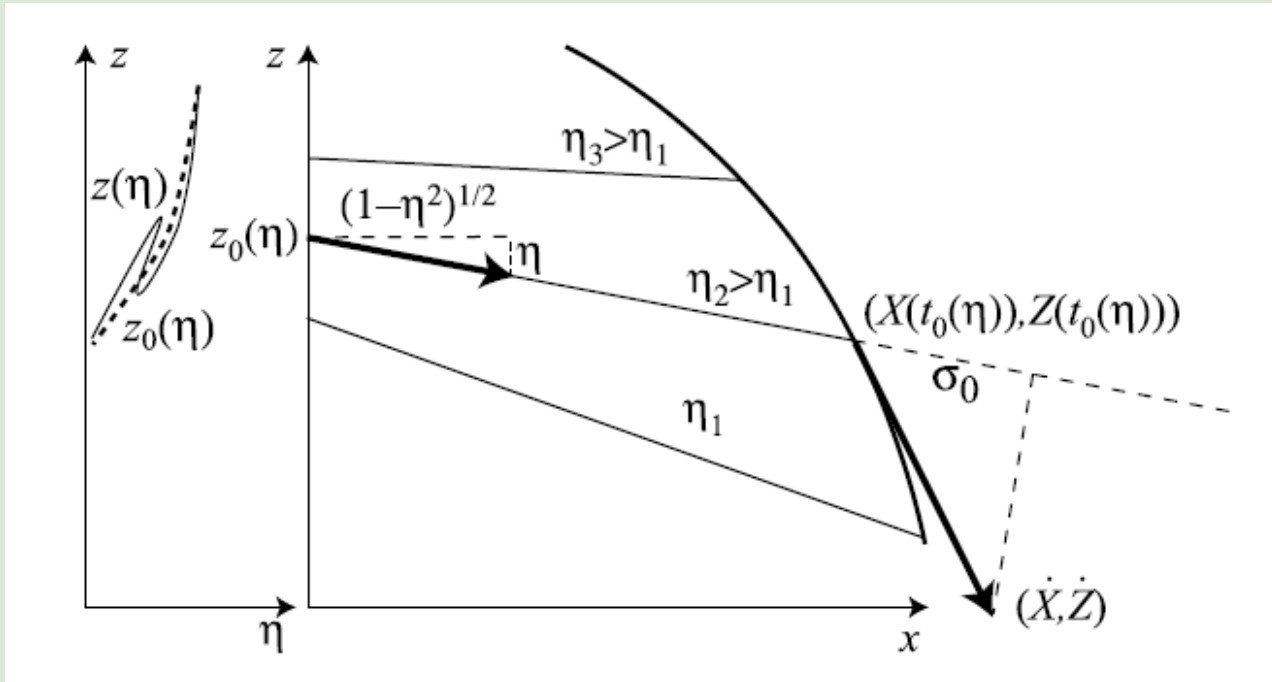
AFM:  
Asymptotic  
forward  
modelling

Diffractive integral (DI):

$$u(t) = \sqrt{\frac{ik}{2\pi}} \int u_0(z) \cos \varphi(z, t) \frac{\exp(ikr(z, t))}{\sqrt{r(z, t)}} dz.$$

# Propagation from the Last Screen

FIO2  
operator  
linearized:



Linearized Zverev Transform (LZT):

$$u(t) = \sqrt{\frac{ik}{2\pi}} \exp\left(ik \int g(t) dt\right) \times \int \exp(ikt\xi) \frac{\exp(-ik \int f(\xi) d\xi) \tilde{u}_0(\eta(\xi))}{\dot{Z}(t_0(\xi)) - \dot{X}(t_0(\xi)) \frac{\eta(\xi)}{\sqrt{1-\eta(\xi)^2}}} d\xi.$$

# Turbulence Model

Correlation function:

$$\langle N(\mathbf{r}_1)N(\mathbf{r}_2) \rangle = \int \exp(i\mathbf{\kappa}(\mathbf{r}_1 - \mathbf{r}_2)) \Phi_N(\mathbf{\kappa}) d^3\mathbf{\kappa}$$

$N(\mathbf{r})$  - turbulent fluctuations of refractivity

$\Phi_N(\mathbf{\kappa})$  - turbulence spectrum

Kolomogorov – van Kármán power law isotropic spectrum:

$$\Phi_N(\mathbf{\kappa}) = 0.033 C_n^2 \left( \kappa^2 + \kappa_0^2 \right)^{-11/6} \exp\left( -\kappa^2 / \kappa_m^2 \right)$$

$\kappa_0 = 2\pi / L_0$        $L_0$  - external scale

$\kappa_m = 5.92 / \lambda_0$        $\lambda_0$  - internal scale



# Turbulence Model

Generic power-law spectrum with constant anisotropy:

$$\Phi_N(\kappa) = A \left( \kappa_y^2 + \eta \left( \kappa_x^2 + \kappa_z^2 \right) + \kappa_0^2 \right)^{-\mu/2} \exp \left( -\kappa^2 / \kappa_m^2 \right)$$

A. S. Gurvich and V. Kan, *Structure of air density fluctuations from cosmic observations of star scintillations: 1. 3D spectrum model and retrieval of its parameters*, Izvestiya, Oceanic and Atmospheric Physics, 2003, 38, No 3, p. 300-310.

A. S. Gurvich and V. L. Brekhovskikh, *Study of the turbulence and inner waves in the stratosphere based on the observations of stellar scintillations from space: a model of scintillation spectra*, Waves Random Media 11 (2001) 163-181.

$\mu = 5$  for internal gravity waves

$\eta \sim 30 - 100$

# Turbulence Model

Optical path fluctuations: 
$$\Psi(x, \Delta x, y, z) = \int_x^{x+\Delta x} N(x', y, z) dx'$$

$$\begin{aligned} \langle \Psi(x, \Delta x, y', z') \Psi(x, \Delta x, y'', z'') \rangle &= \\ &= \int \exp(i\mu_y (y' - y'') + i\mu_z (z' - z'')) \Phi_\Psi(\boldsymbol{\mu}) d\mu_y d\mu_z \end{aligned}$$

$$\Phi_\Psi(\boldsymbol{\mu}) = \Delta x^2 \int \text{sinc}^2\left(\frac{\Delta x \kappa_x}{2}\right) \Phi_N(\kappa_x, \mu_y, \mu_z) d\kappa_x$$

Thick layer approximation:

$$\Phi_\Psi(\boldsymbol{\mu}) = 2\pi \Delta x \Phi_N(0, \mu_y, \mu_z)$$

$$\Phi_\Psi^{1D}(\mu_z) = \int \Phi_\Psi(\boldsymbol{\mu}) d\mu_y$$

# WOP simulation results

ECMWF: local N-profile

COSMIC: BA, N obtained from CT2-retrieval of COSMIC data

WOP: Simulation with ECMWF field without turbulence; BA, N obtained from CT2

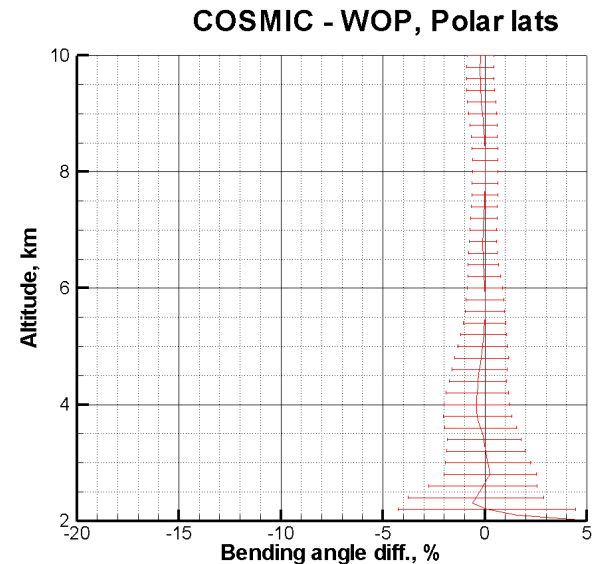
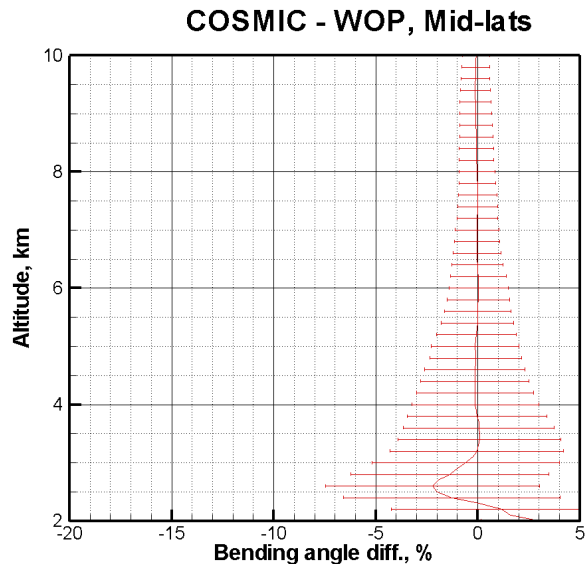
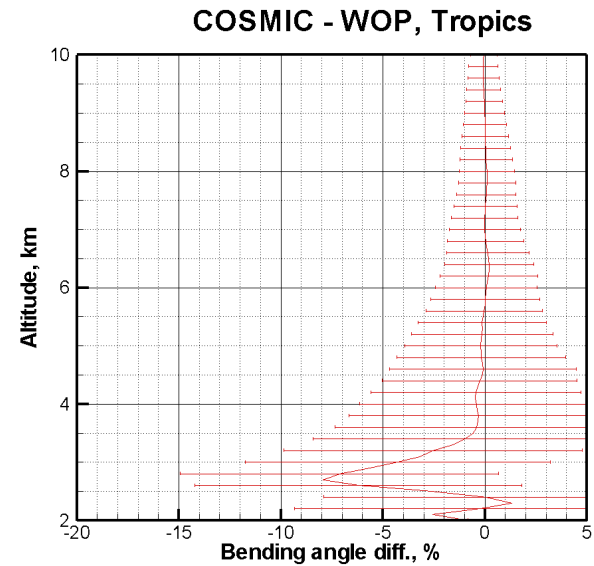
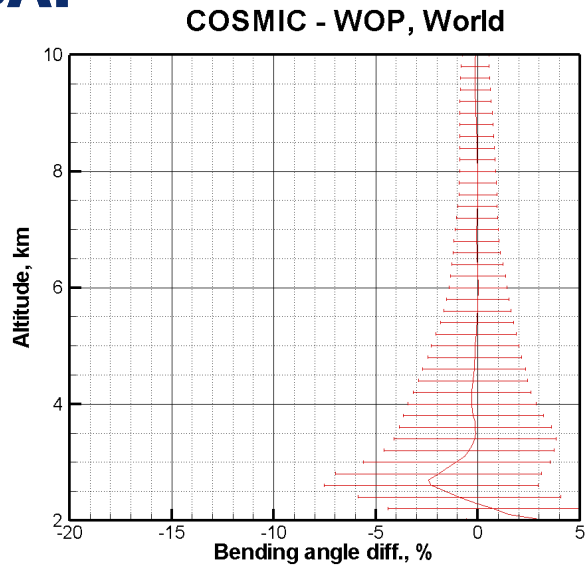
WOP\_turb: Simulation with turbulence; BA, N obtained from CT2

a) weak turbulence

b) strong turbulence (twice as strong as (a))

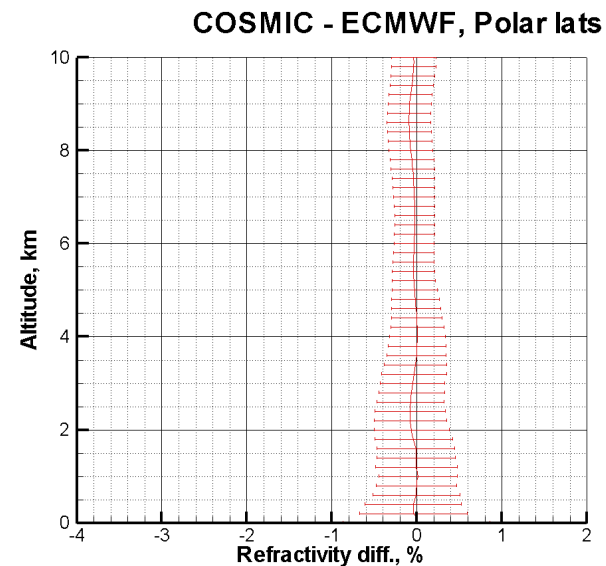
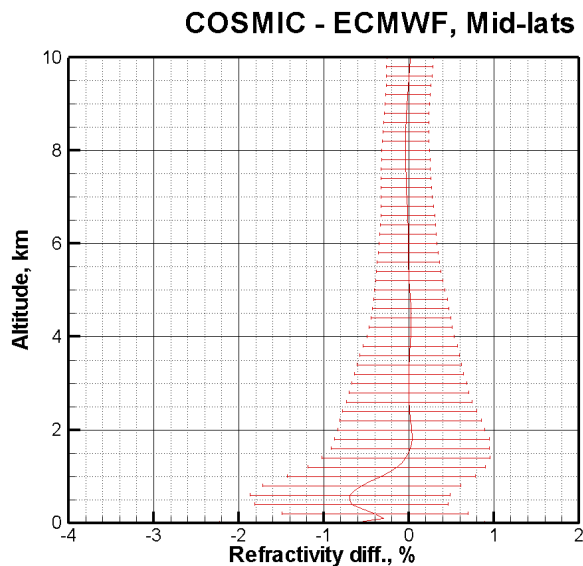
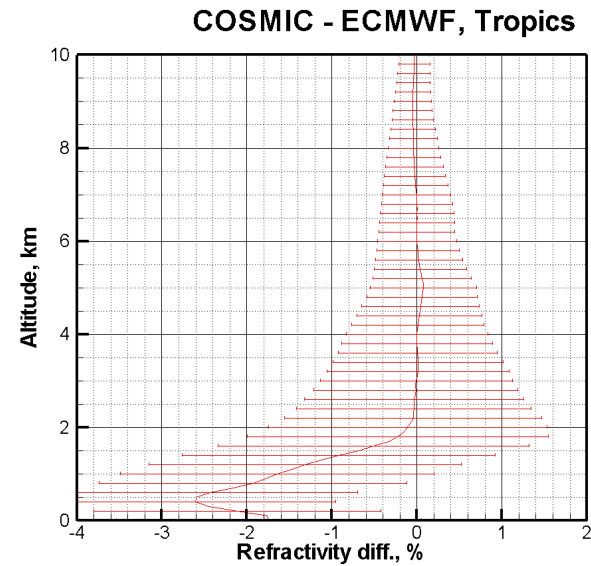
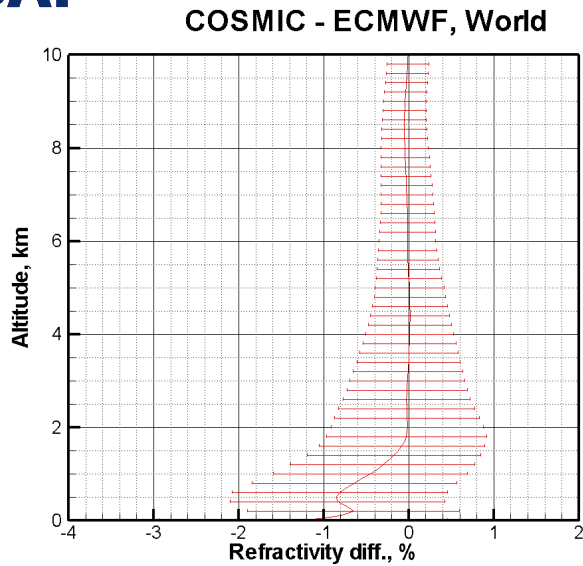
# Strong turbulence

1)



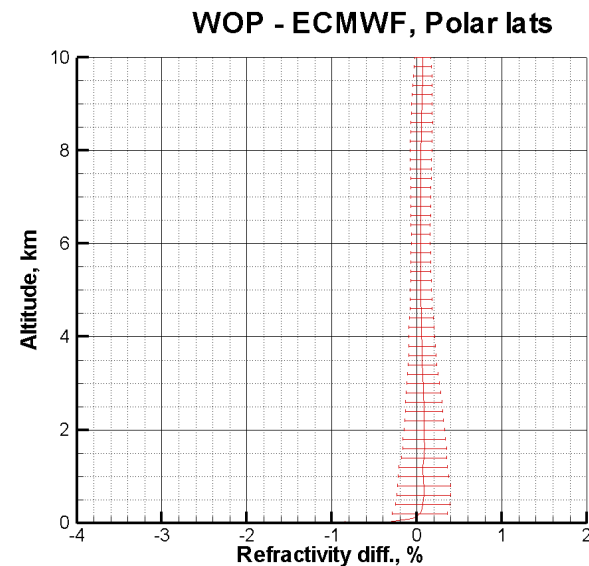
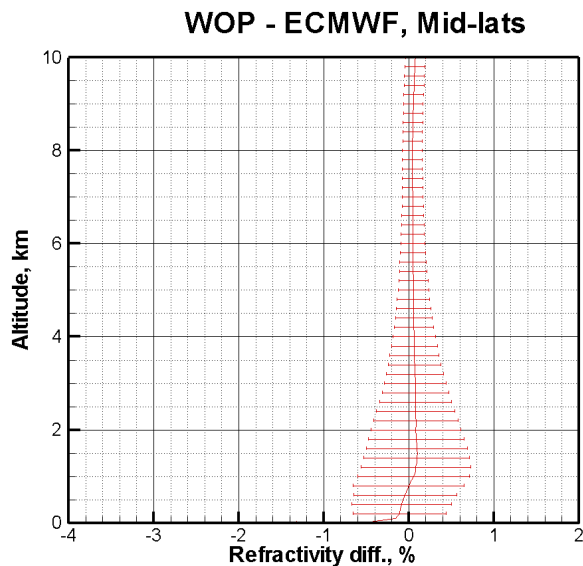
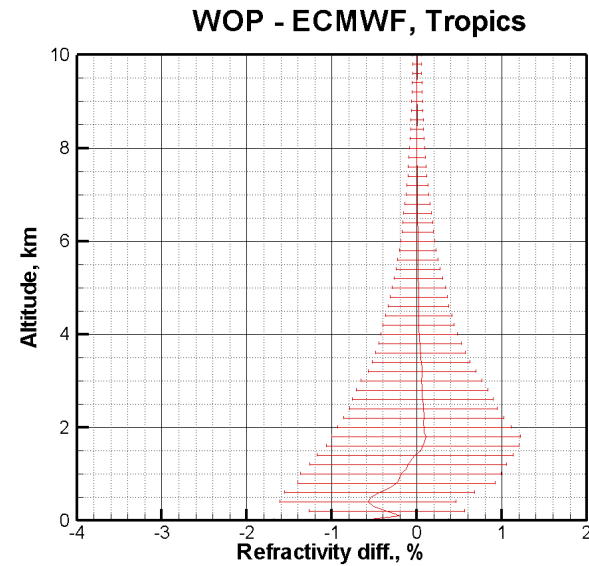
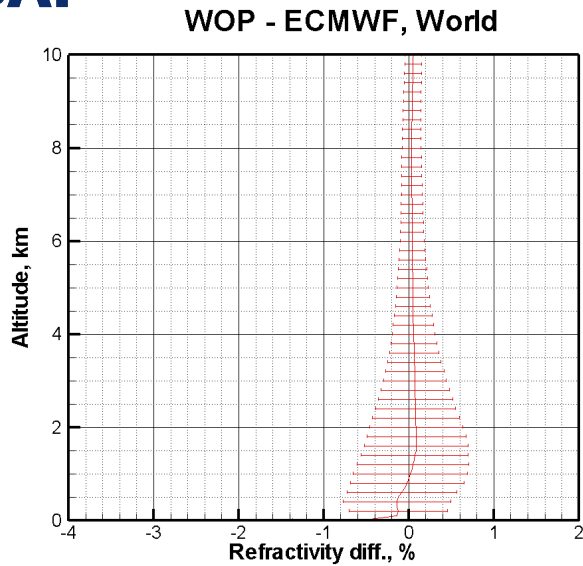
# Strong turbulence

2)



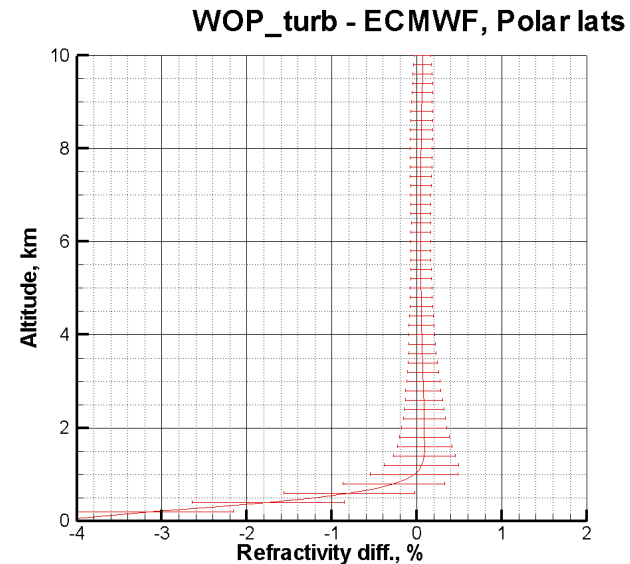
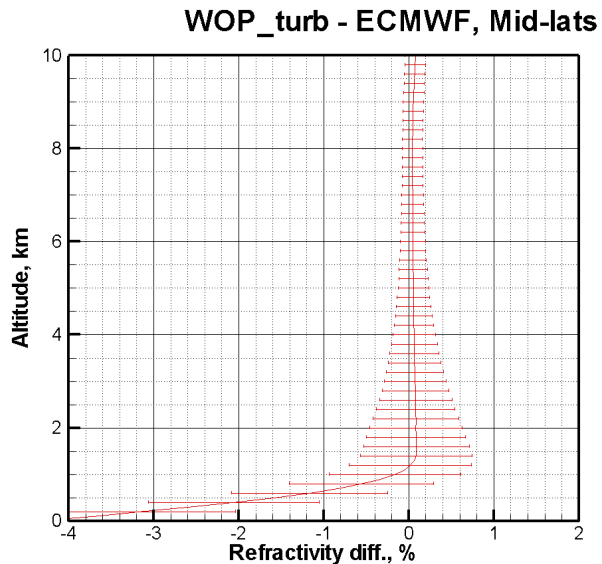
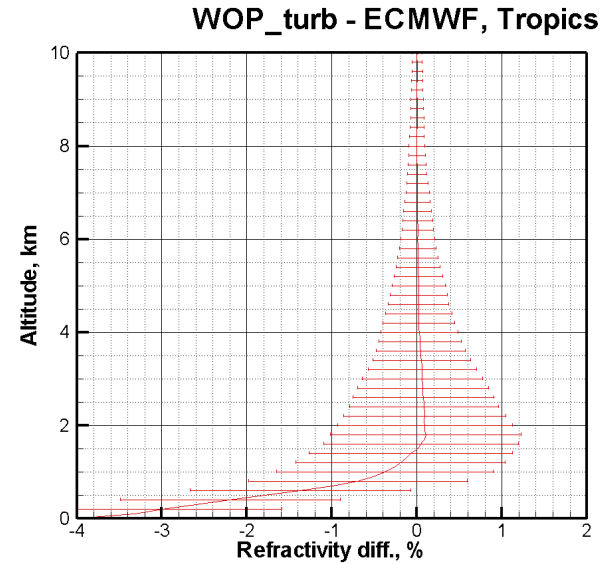
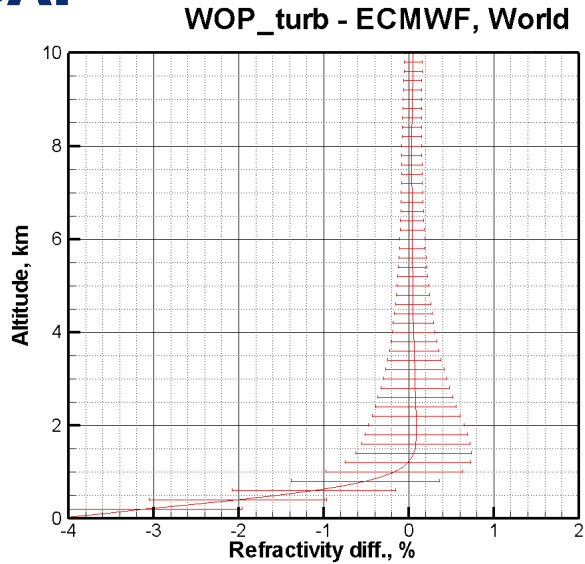
# Strong turbulence

3)



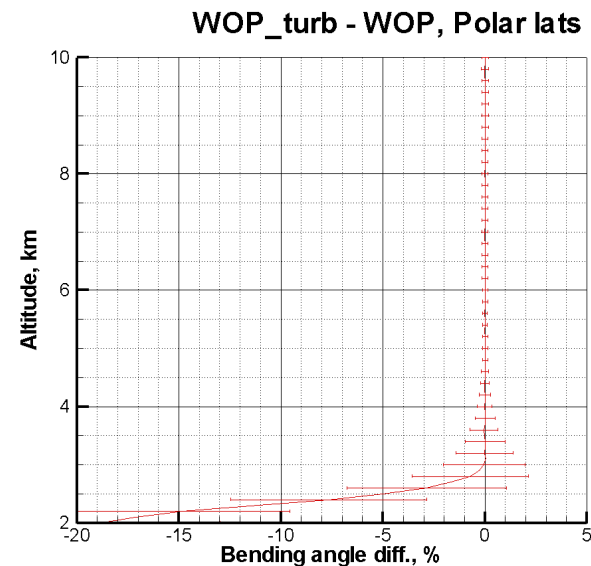
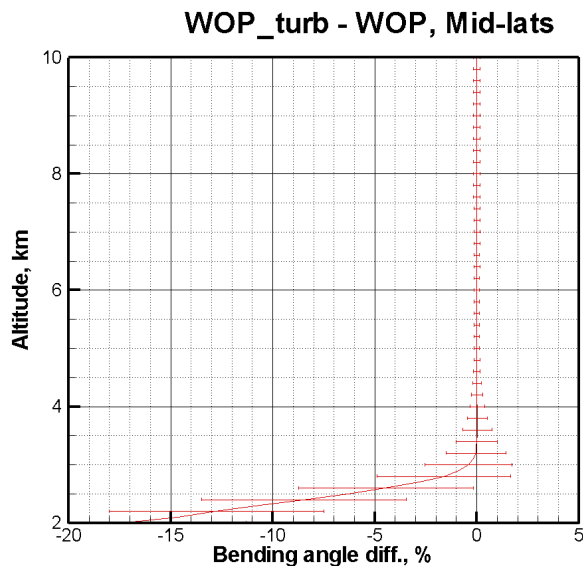
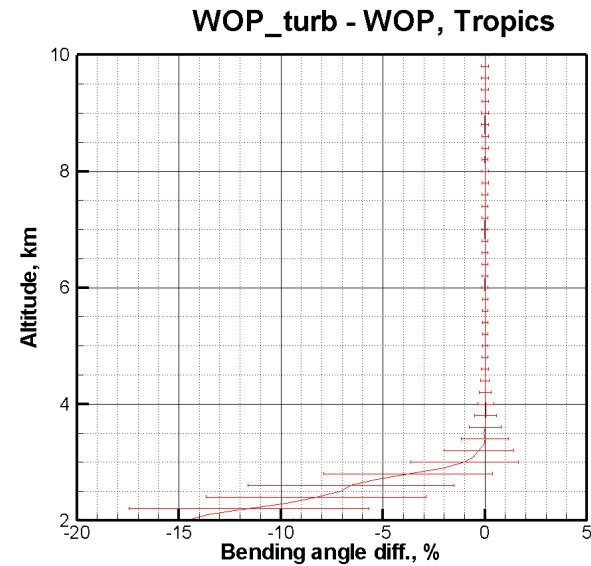
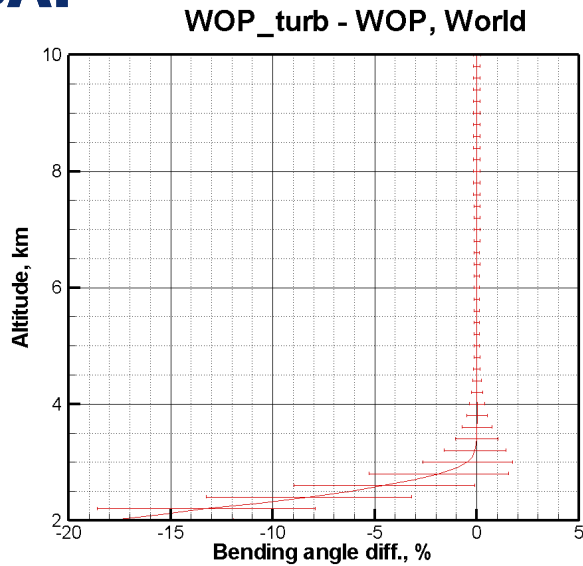
# Strong turbulence

4)



# Strong turbulence

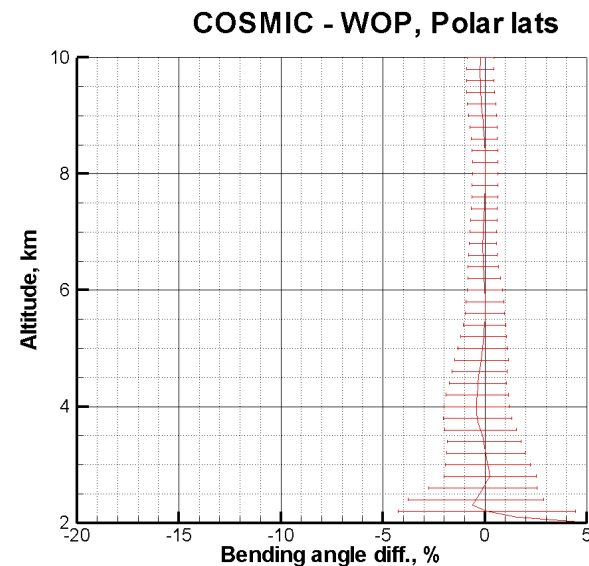
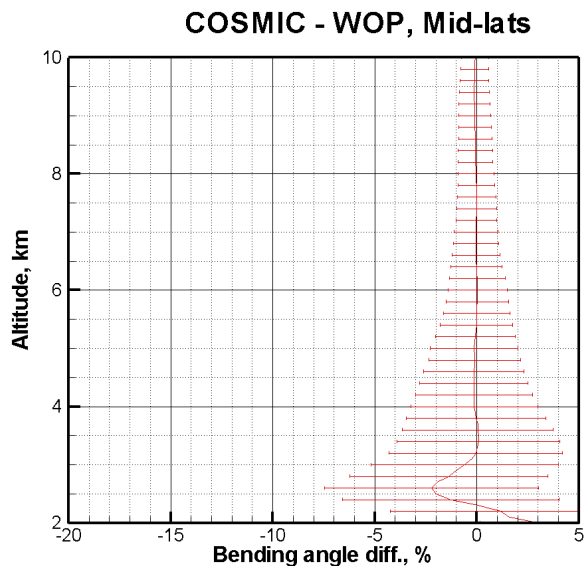
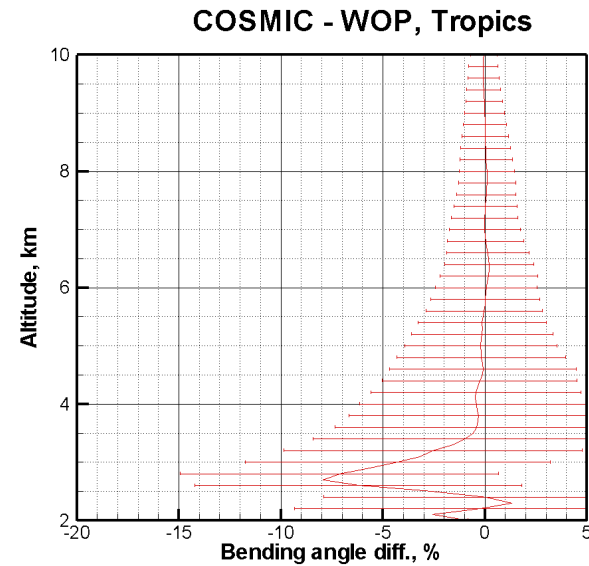
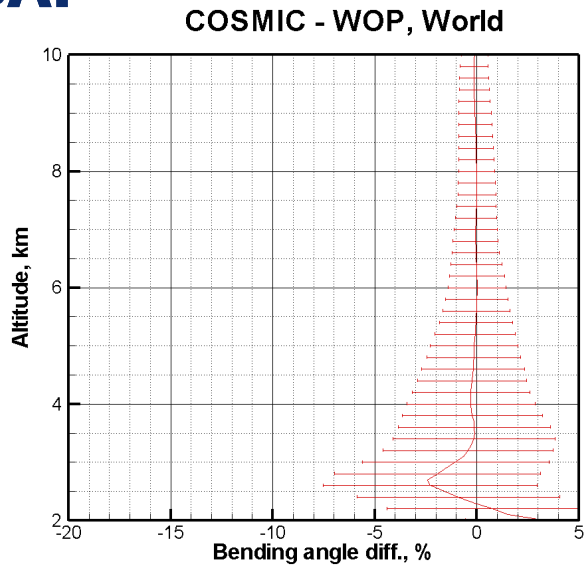
5)





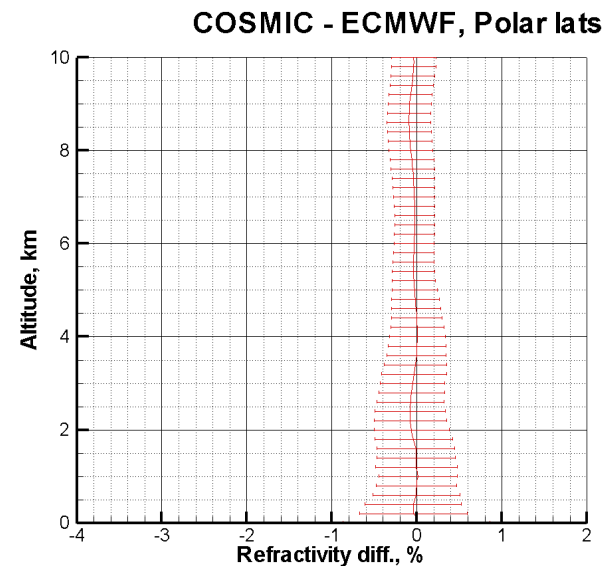
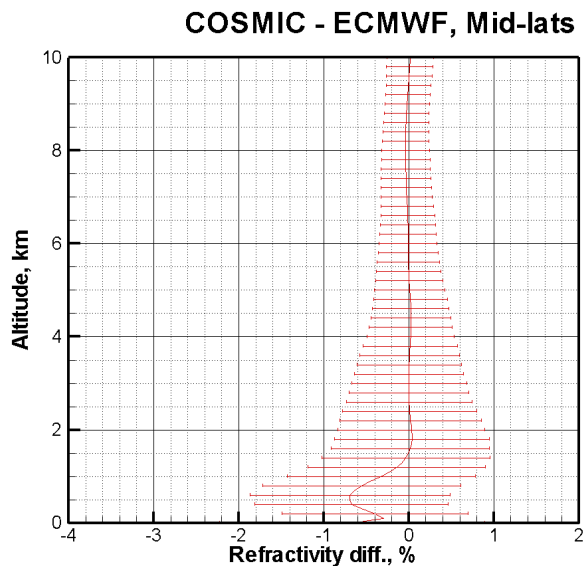
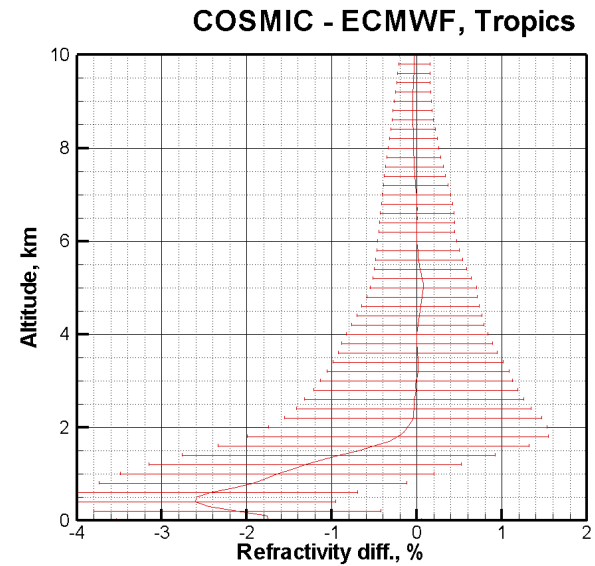
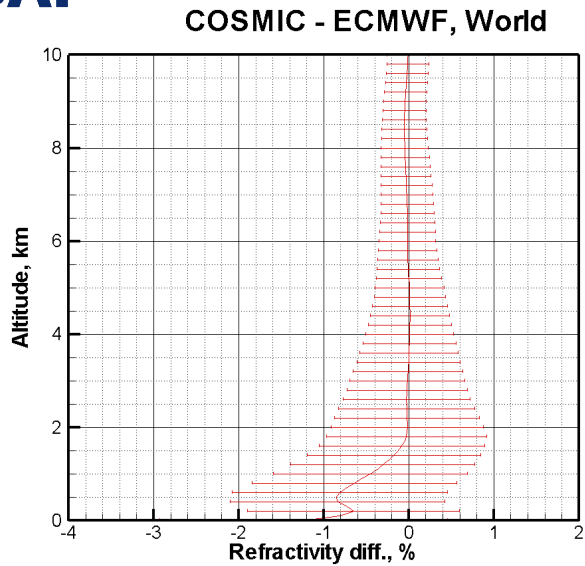
# Weak turbulence

1)



# Weak turbulence

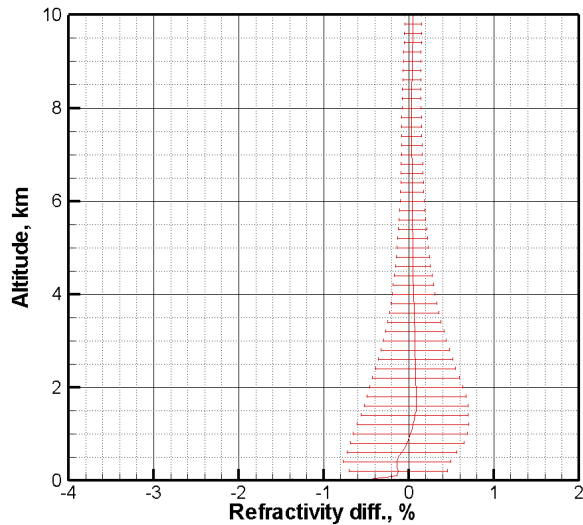
2)



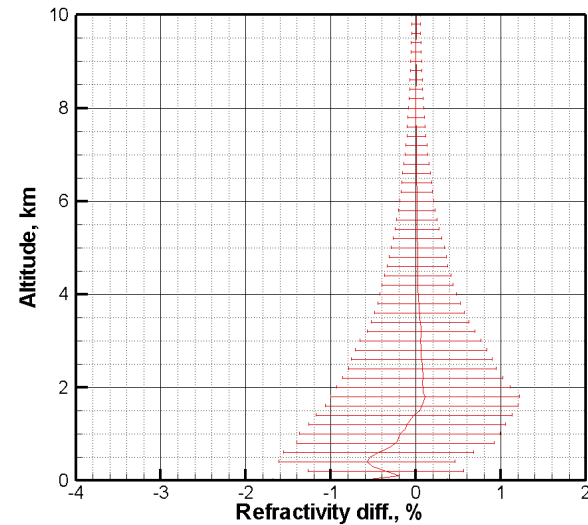
# Weak turbulence

3)

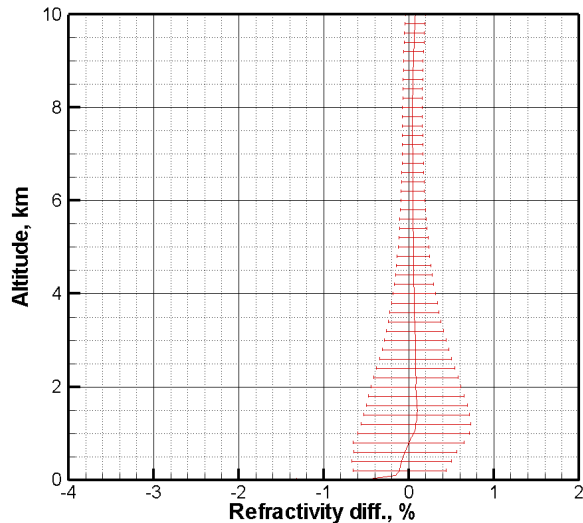
WOP - ECMWF, World



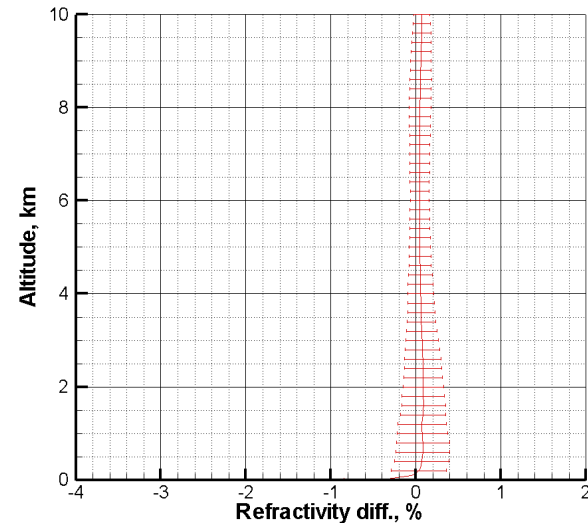
WOP - ECMWF, Tropics



WOP - ECMWF, Mid-lats

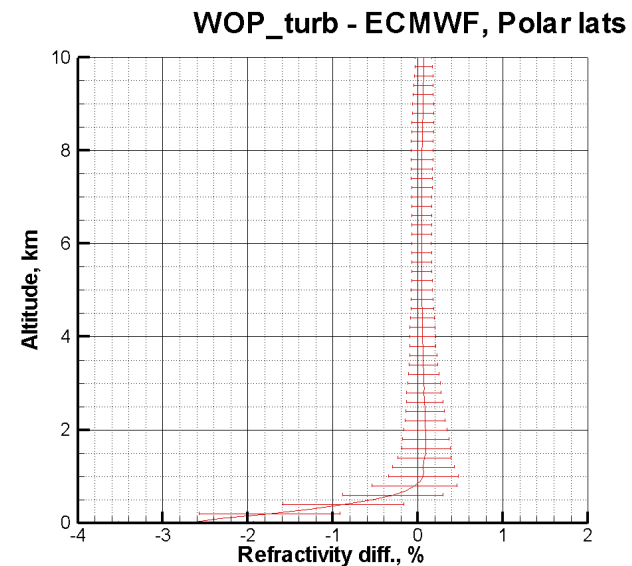
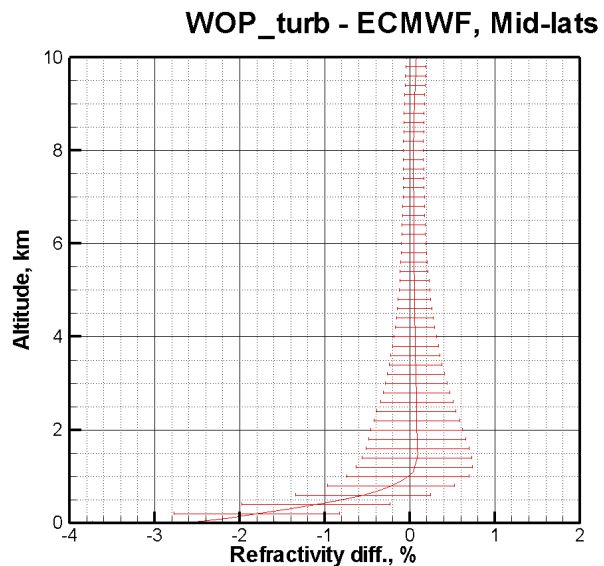
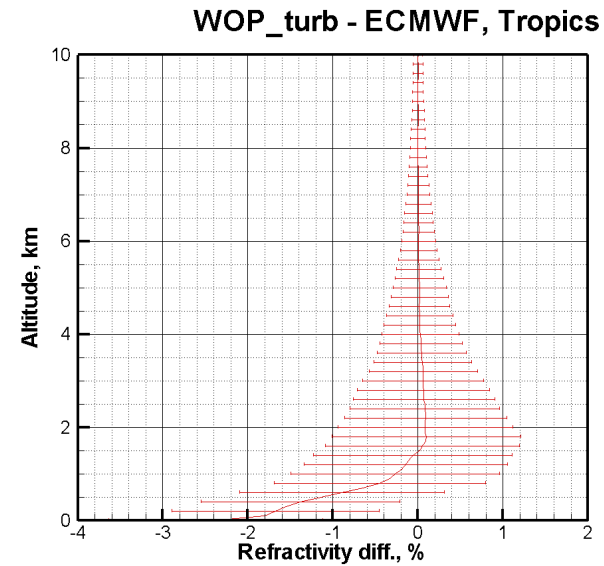
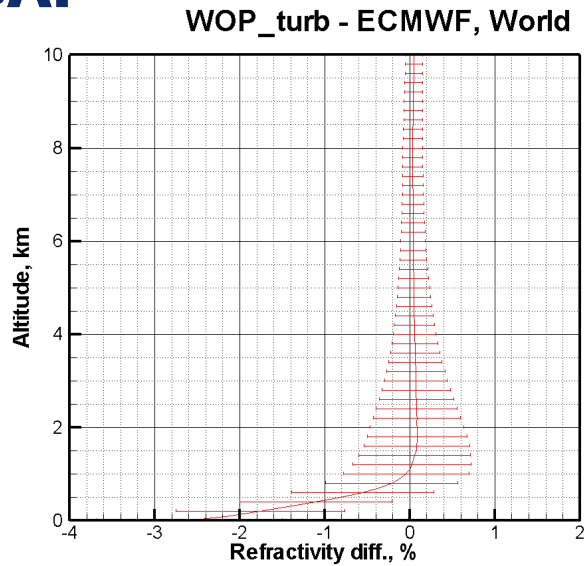


WOP - ECMWF, Polar lats



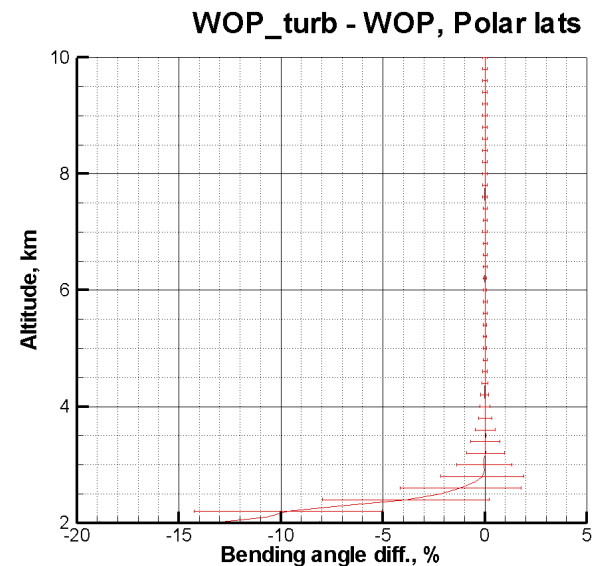
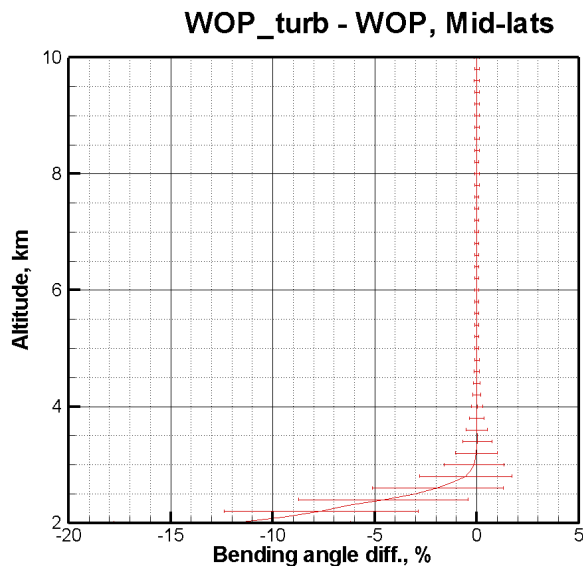
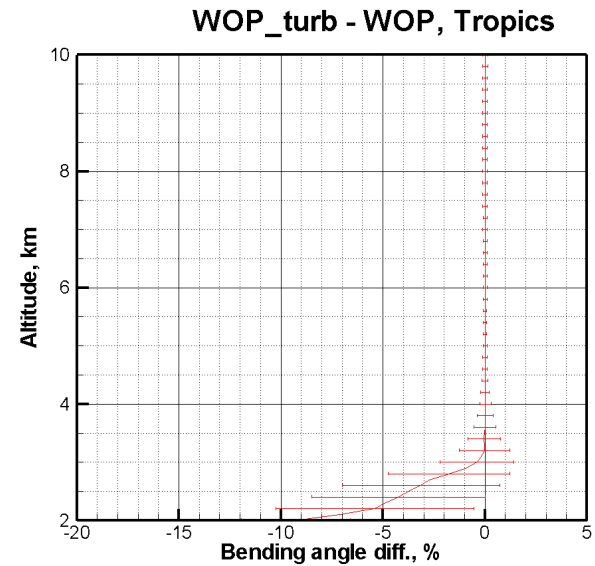
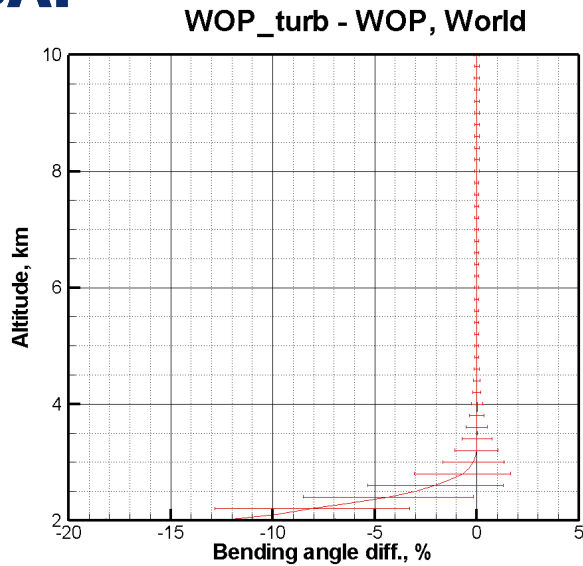
# Weak turbulence

4)

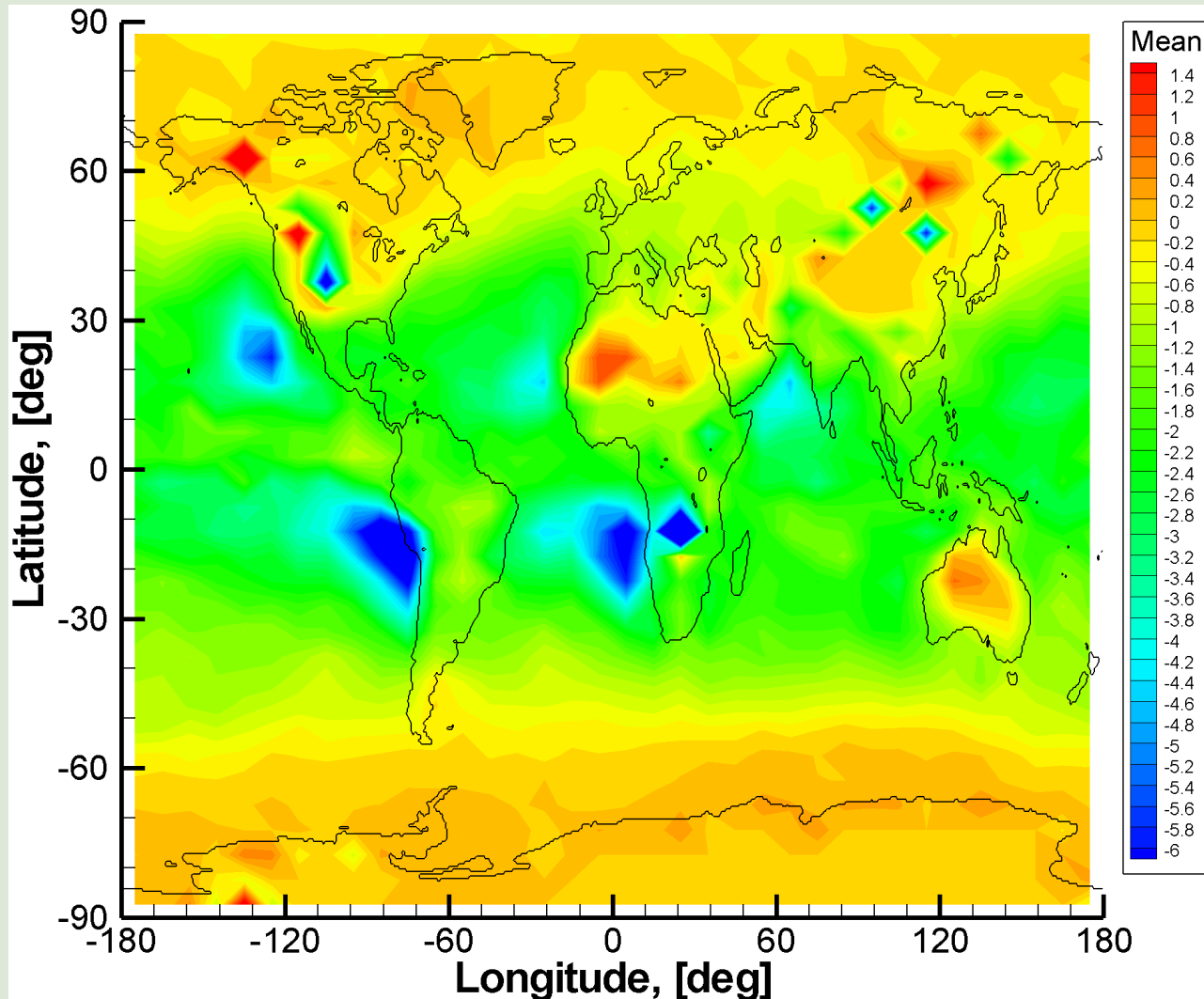


# Weak turbulence

5)

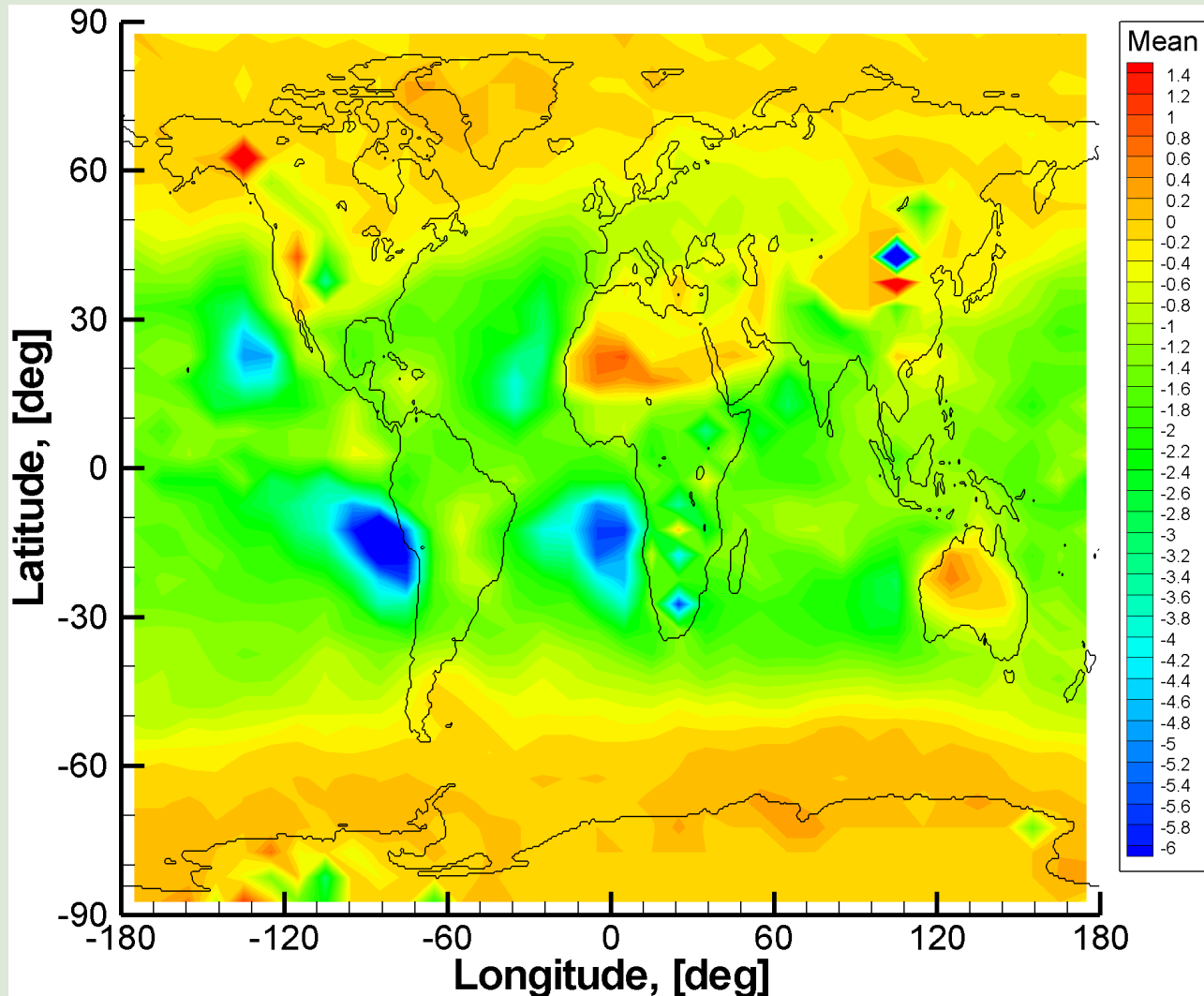


# COSMIC Statistics



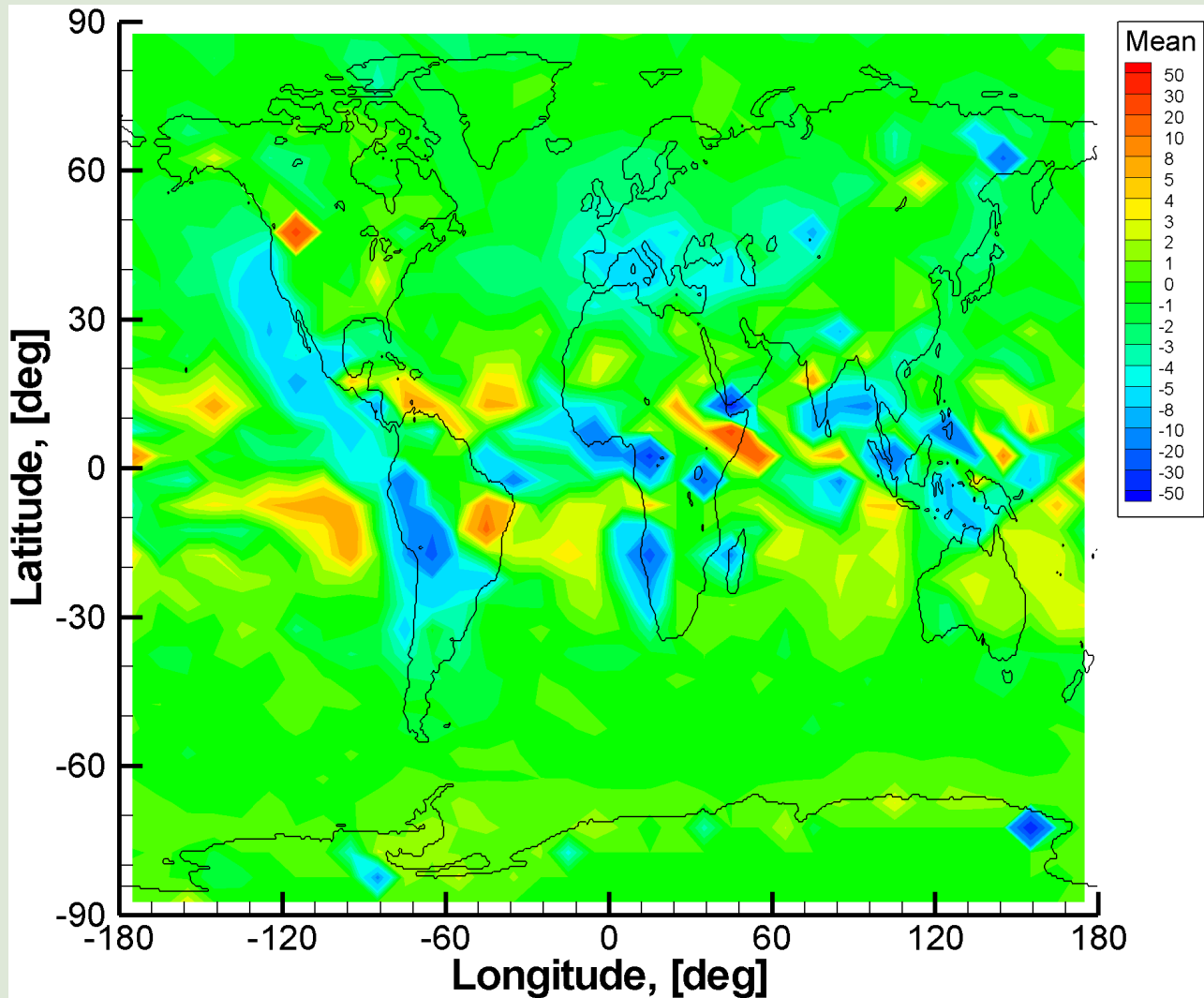
COSMIC-ECMWF refractivities (O-B/B, %) at an altitude of 0.6 km

# COSMIC Statistics

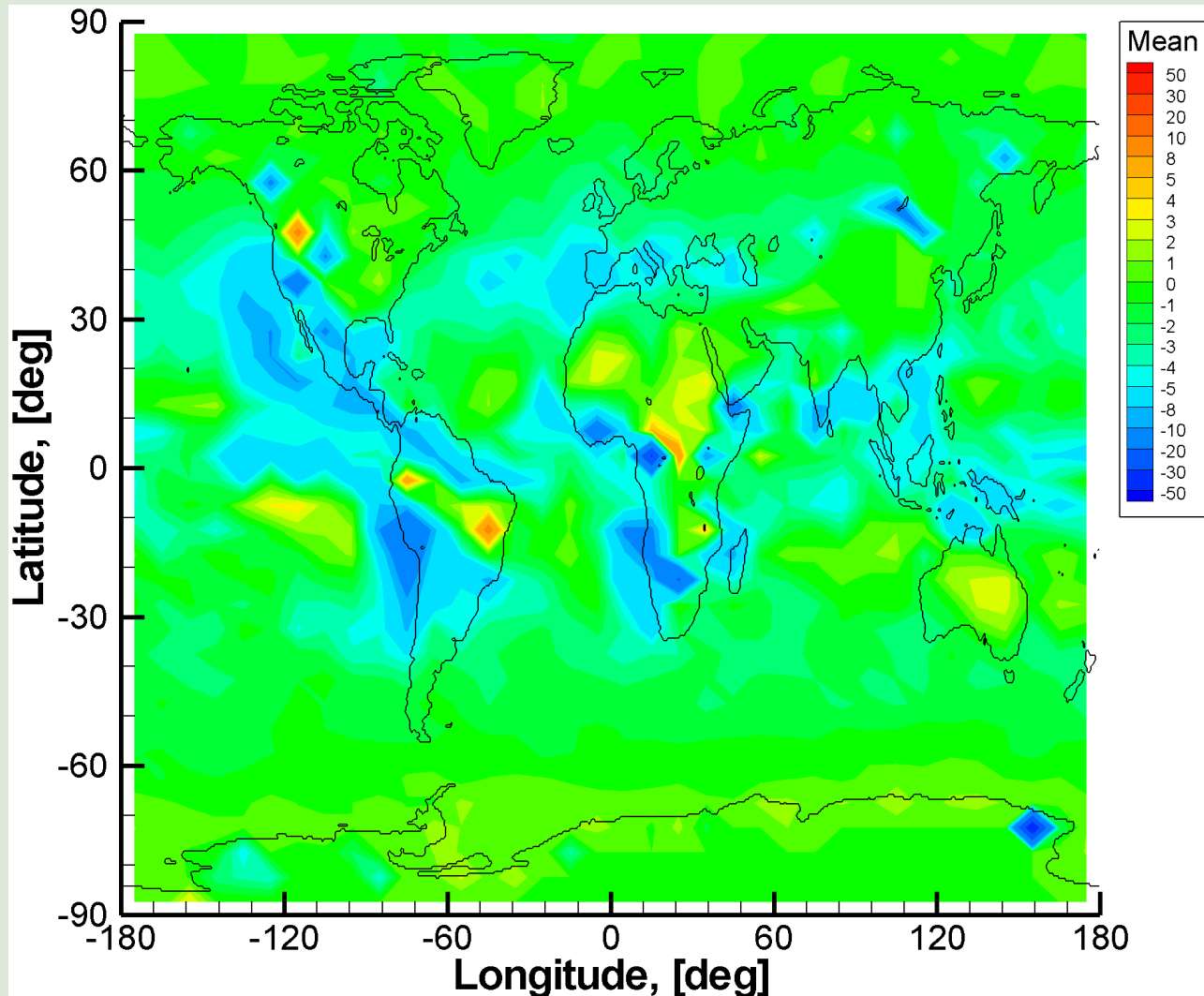


COSMIC-ECMWF refractivities (O-B/B, %) at an altitude of 0.8 km

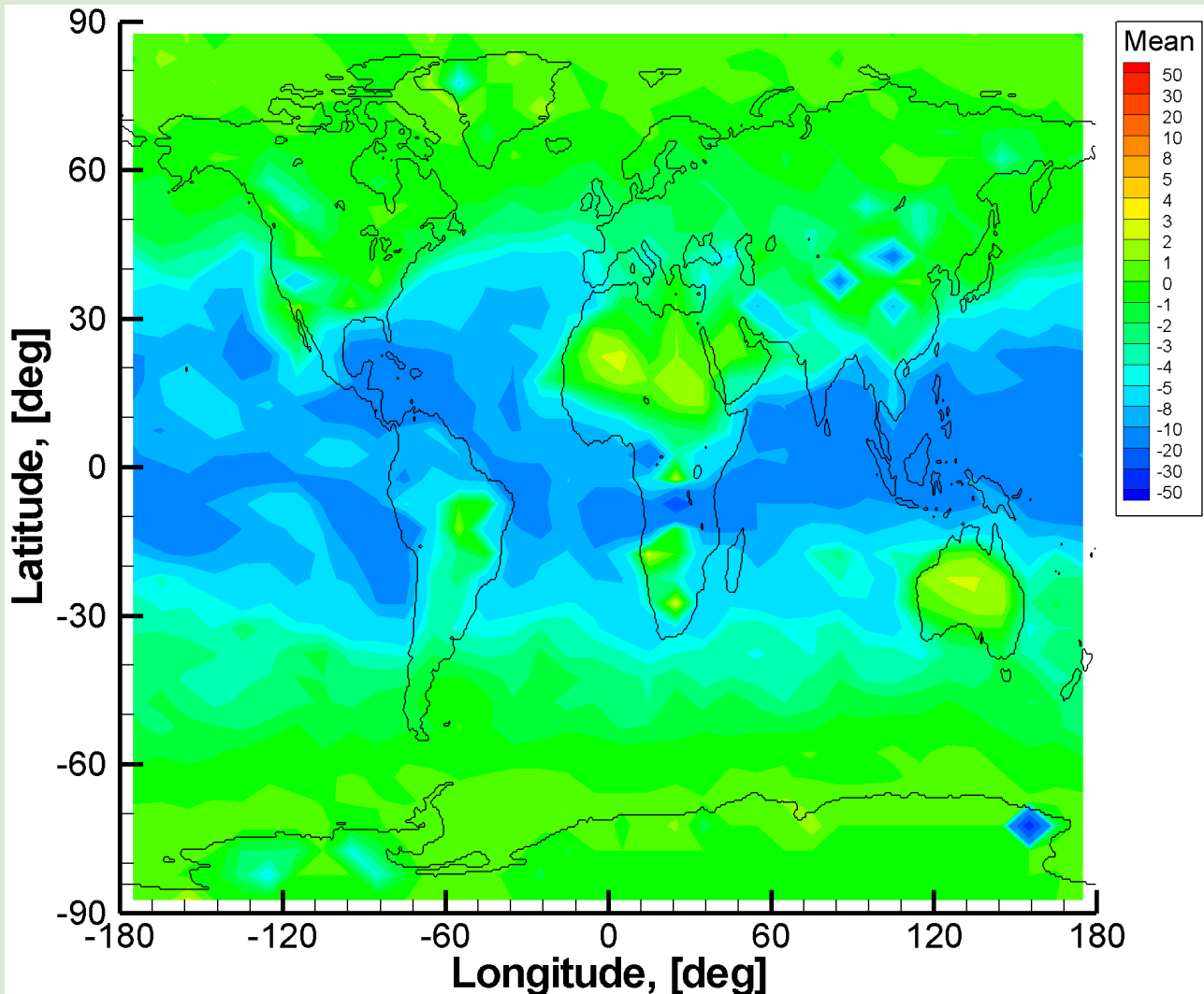
# COSMIC Statistics



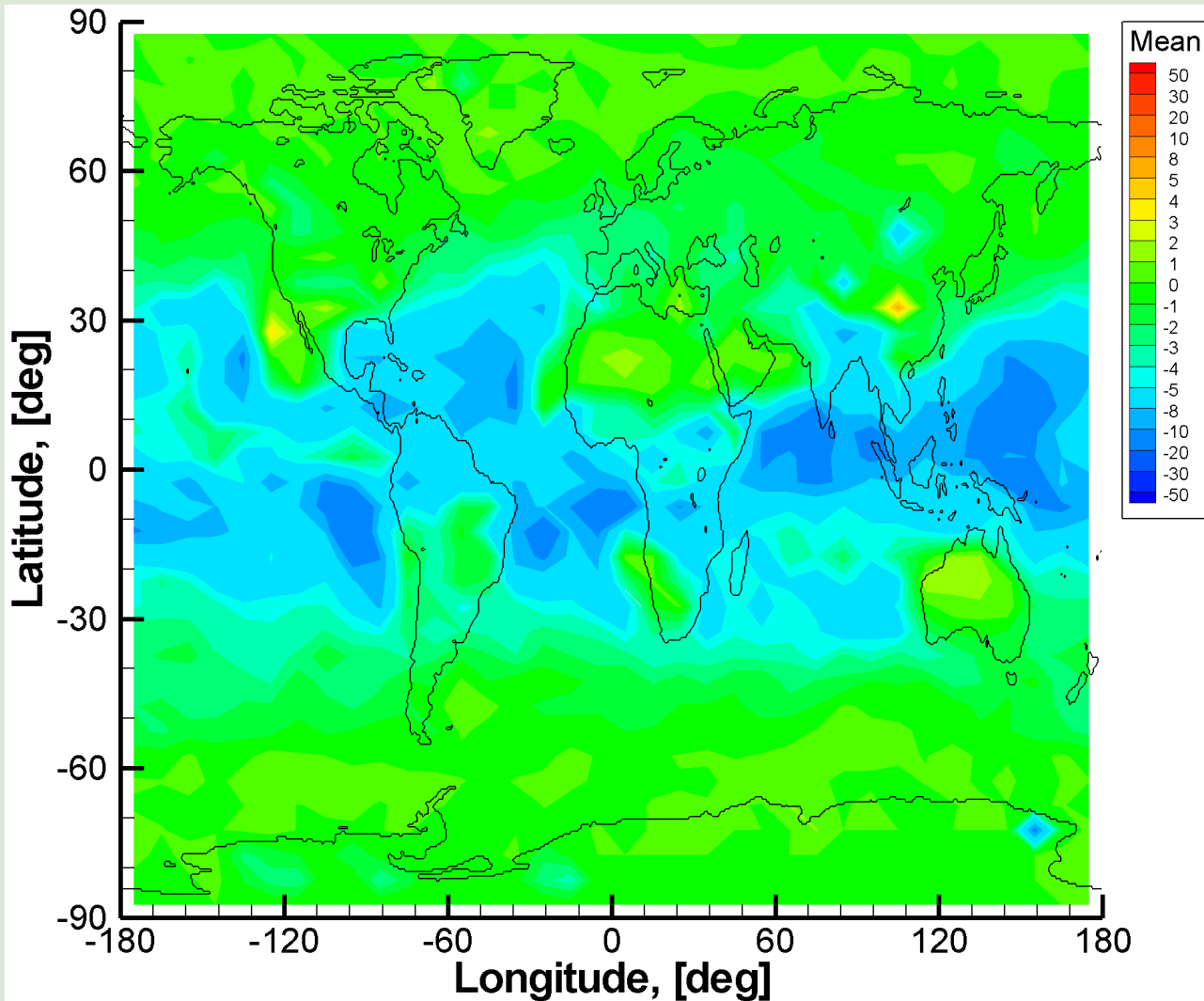




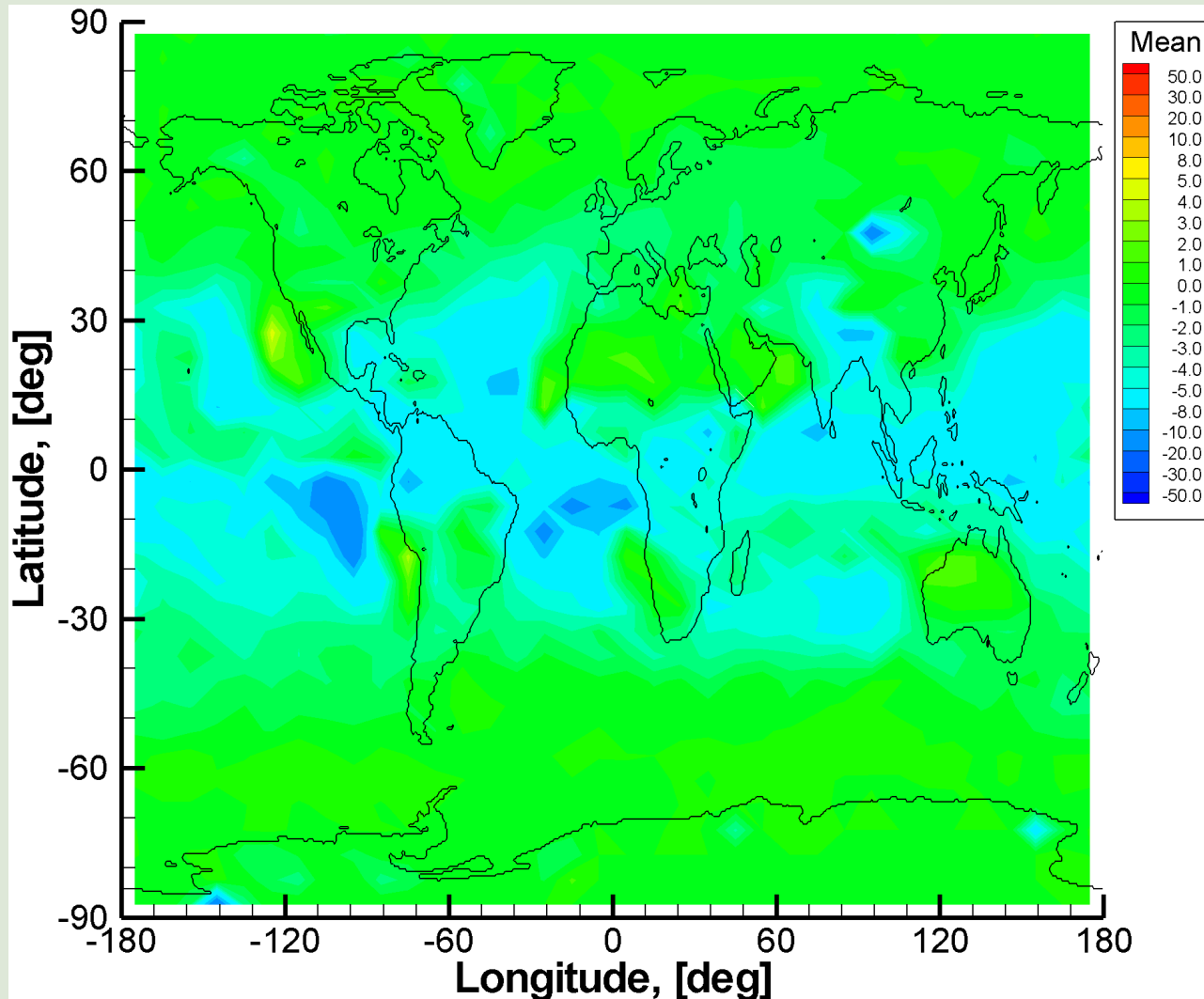
# COSMIC Statistics



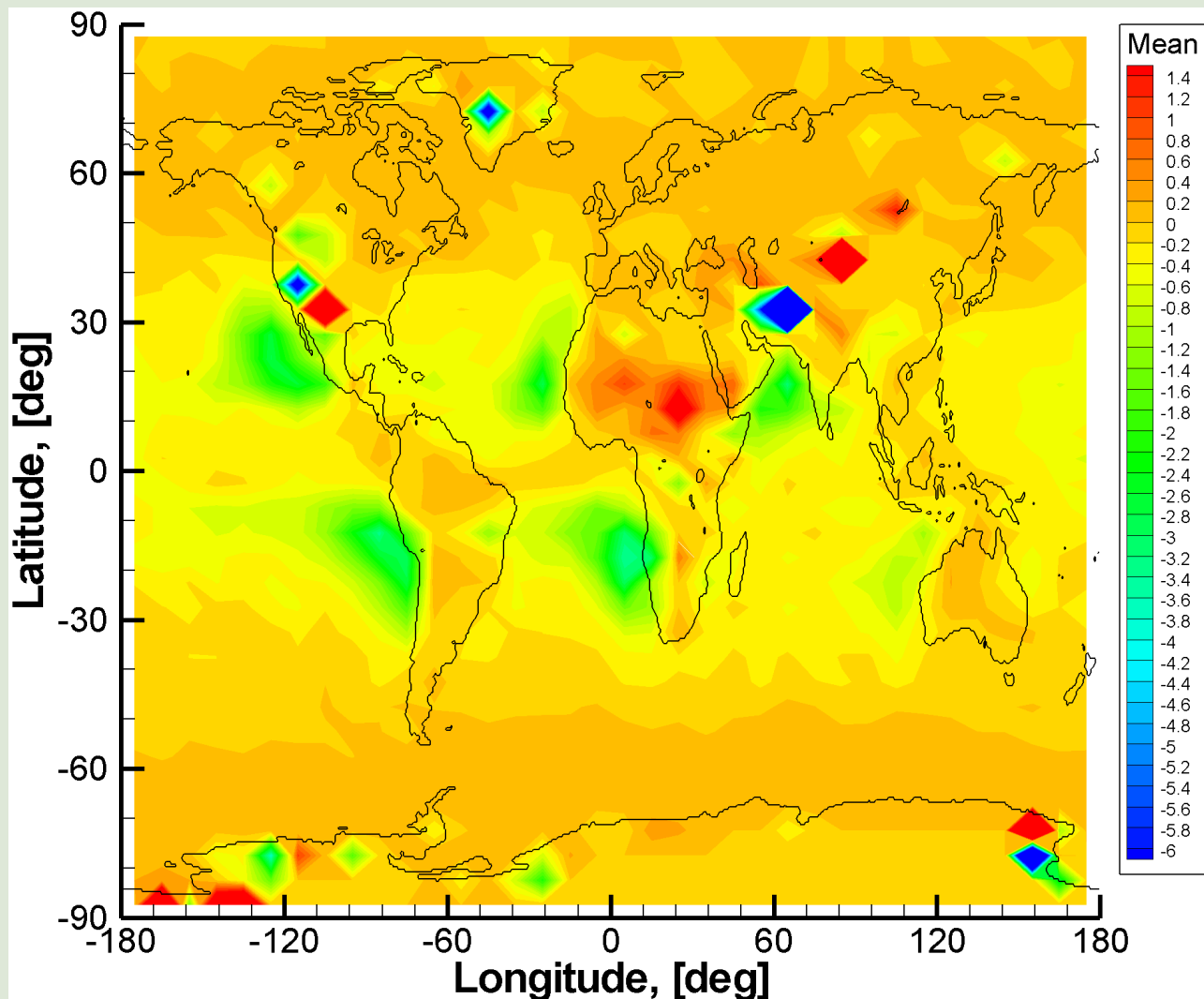
# COSMIC Statistics

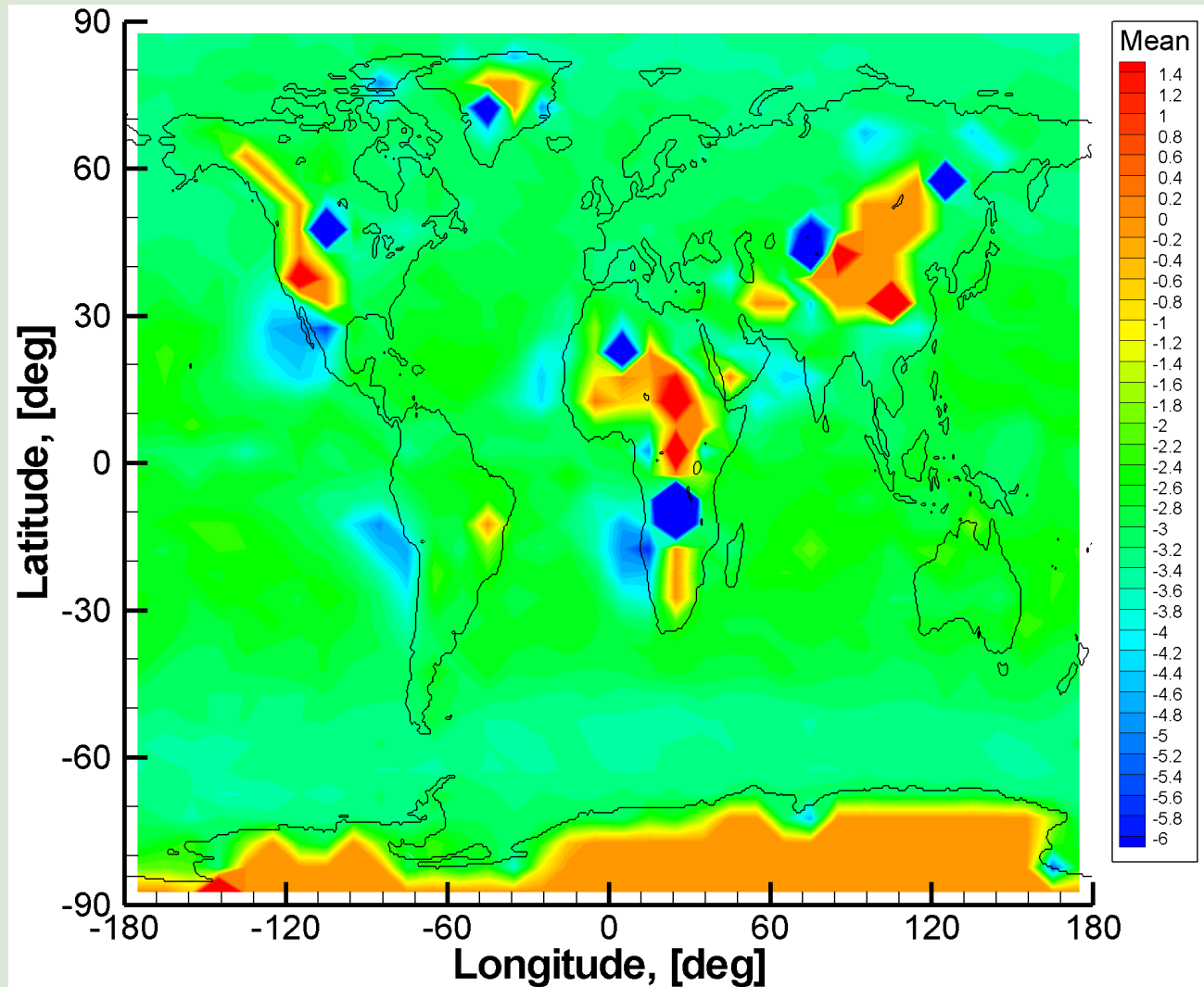


COSMIC-ECMWF bending angles (O-B/B, %) at an impact height of 2.9 km



# ECMWF-based Simulation





# Conclusions

1. Fluctuations as one of the sources of systematic RO retrievals error has been considered early by Eshleman and Haugstad (1977).
2. A simple theoretical explanation of this effect is based on the Fermat principle. However, accurate theoretical estimates for decimeter radio waves taking into account regular refraction and realistic fluctuation levels are not straightforward.
3. To arrive at estimates of this effect we performed numerical simulations by the phase screen method with a fluctuating component in the phase screen optical paths.
4. The fluctuations we are considering are not necessarily turbulence. They belong to a larger scale range. We use an effective Kolmogorov – van Kármán spectrum with parameters tuned in such a way to comply with typical amplitude fluctuation strength.
5. We show that it is possible to obtain realistic magnitudes of the N-bias in the simulated retrievals.



# Acknowledgments

This work was supported by a Visiting Scientist activity by the Radio Occultation Meteorology Satellite Application Facility (ROM SAF) and by the Russian Foundation for Basic Research (grant 12-05-00335-a)