Formation-Flying CubeSat Constellations for Internal Gravity Wave Tomography

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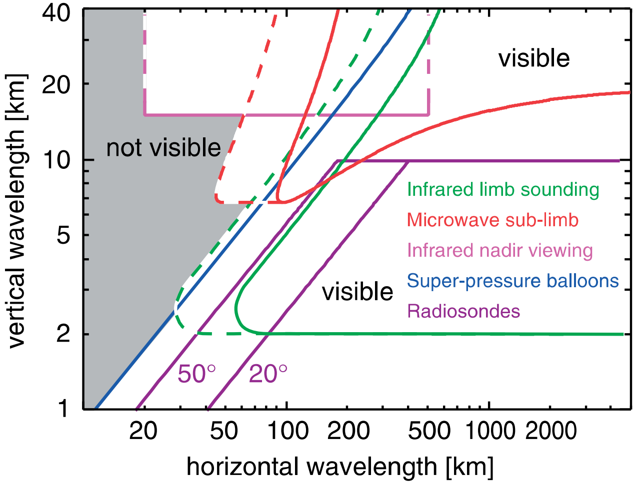
Outline

- Formation Flying for Clustered Occultations
- Occultation Cluster Quality and Distribution
- Constellation Maintenance and Lifetime
- Conclusions

Formation Flight & Sounding Clusters

Previous Work: RO Tomography of Gravity Waves

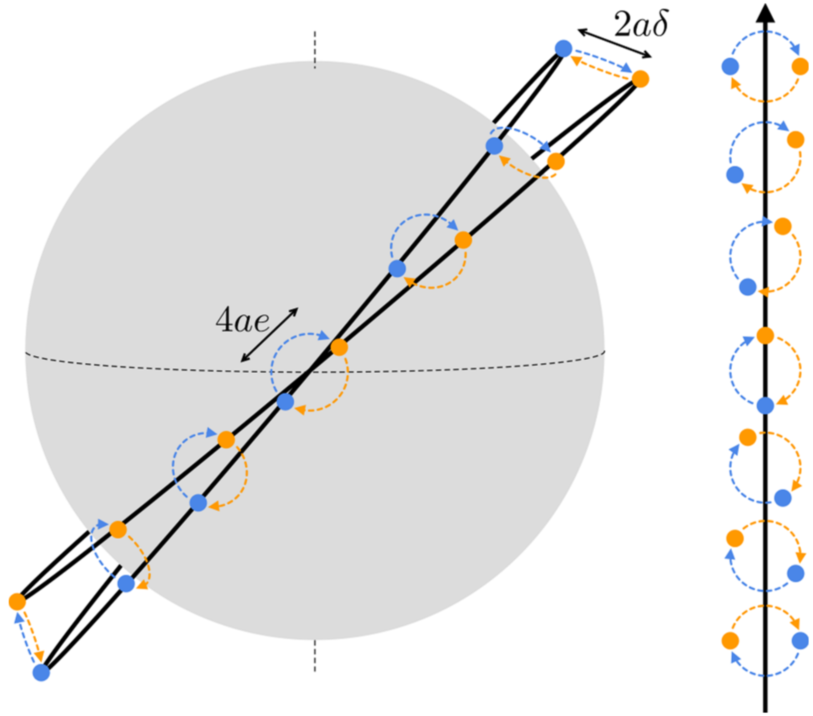
- Technique demonstrated with COSMIC-1 data, before satellites dispersed: Wang and Alexander, JGR (2010); Schmidt, Alexander, de la Torre (2016). Sensitive only to long wavelengths (~1000 km) and low frequencies (~1/day) in order to find any clusters.
- A dedicated mission can find short wavelength (~50 km), high frequency waves (~1/30 min). Otherwise ~10,000 randomly distributed LEO satellites would be needed to develop an RO momentum flux climatology of these waves.

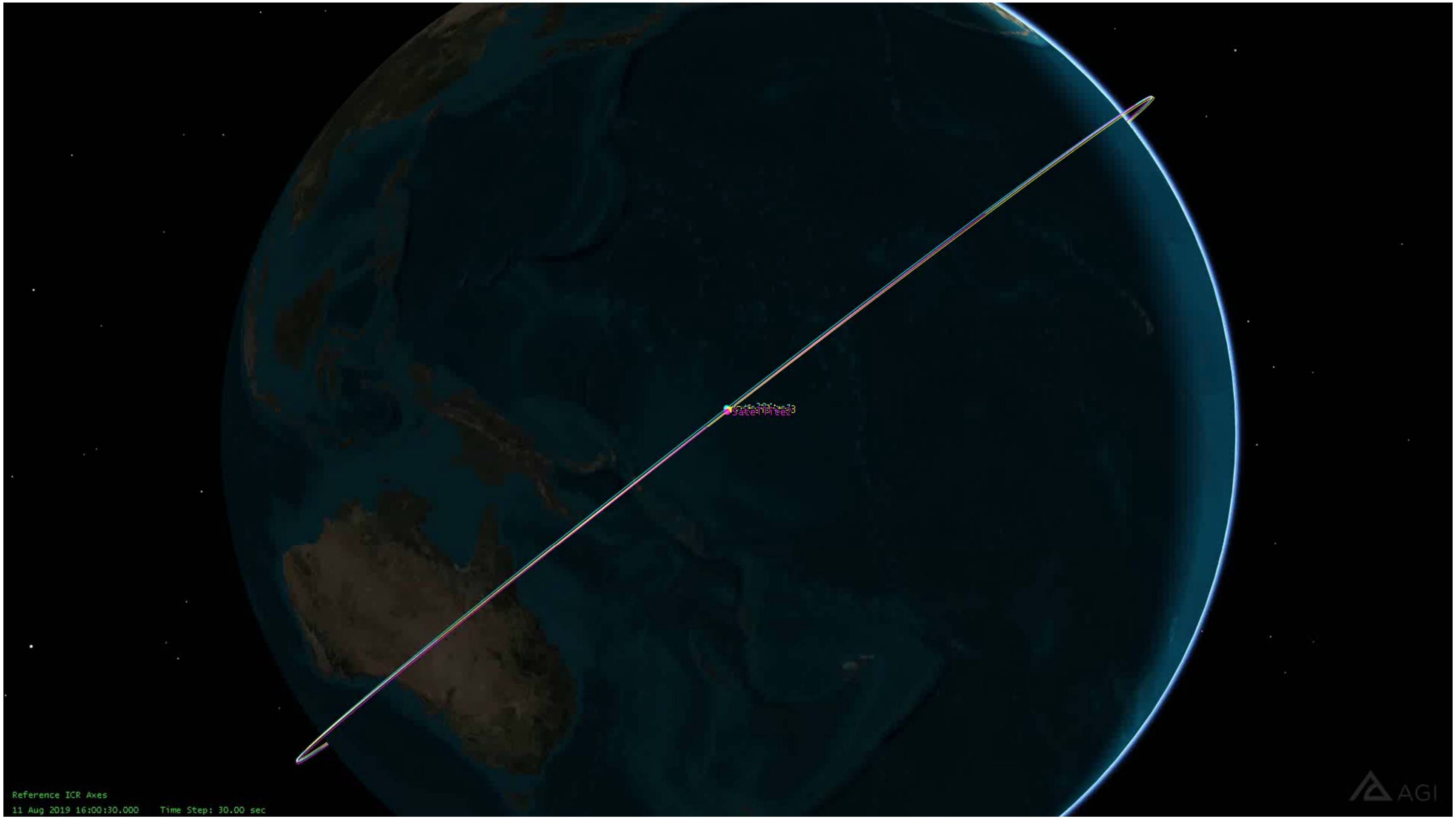


Alexander et al., QJRMS (2010)

Mutual Orbit Groups

- Satellites slightly perturbed in inclination, RAAN, and eccentricity can be made to "orbit" each other, and orbit the Earth as a group.
- "Mutual Orbit Groups" can be constructed in any orbit, with any number of satellites.
- (The orbit angular momentum vectors and eccentricity vectors should be distributed in cones.)




Satellite3
Satellite2
Reference ICR Axes
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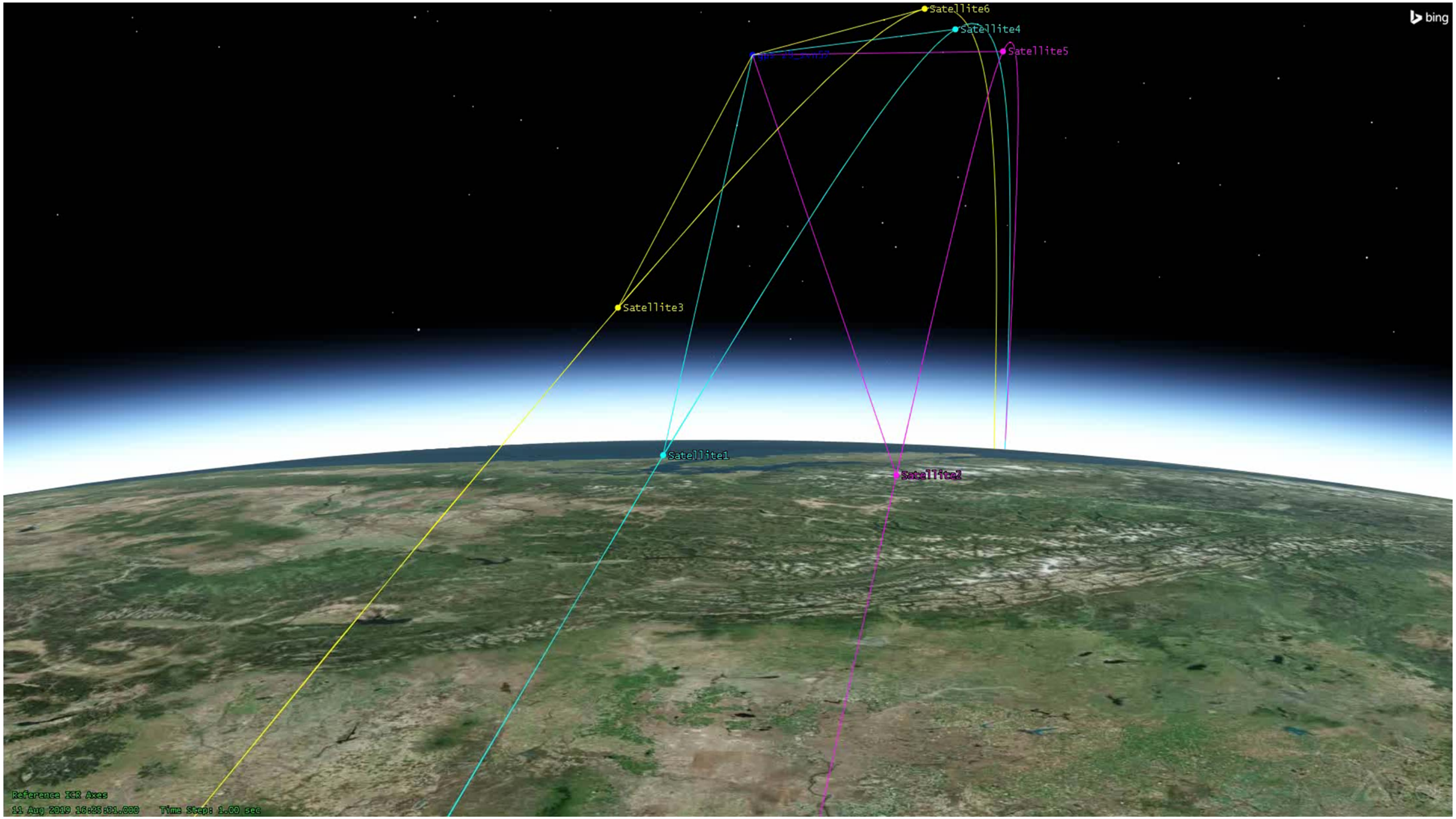
AGI


Satellite6
Satellite4
Satellite5
gps 29_svn57
Satellite3
Satellite1
Satellite2
bing
Reference ICR Axes
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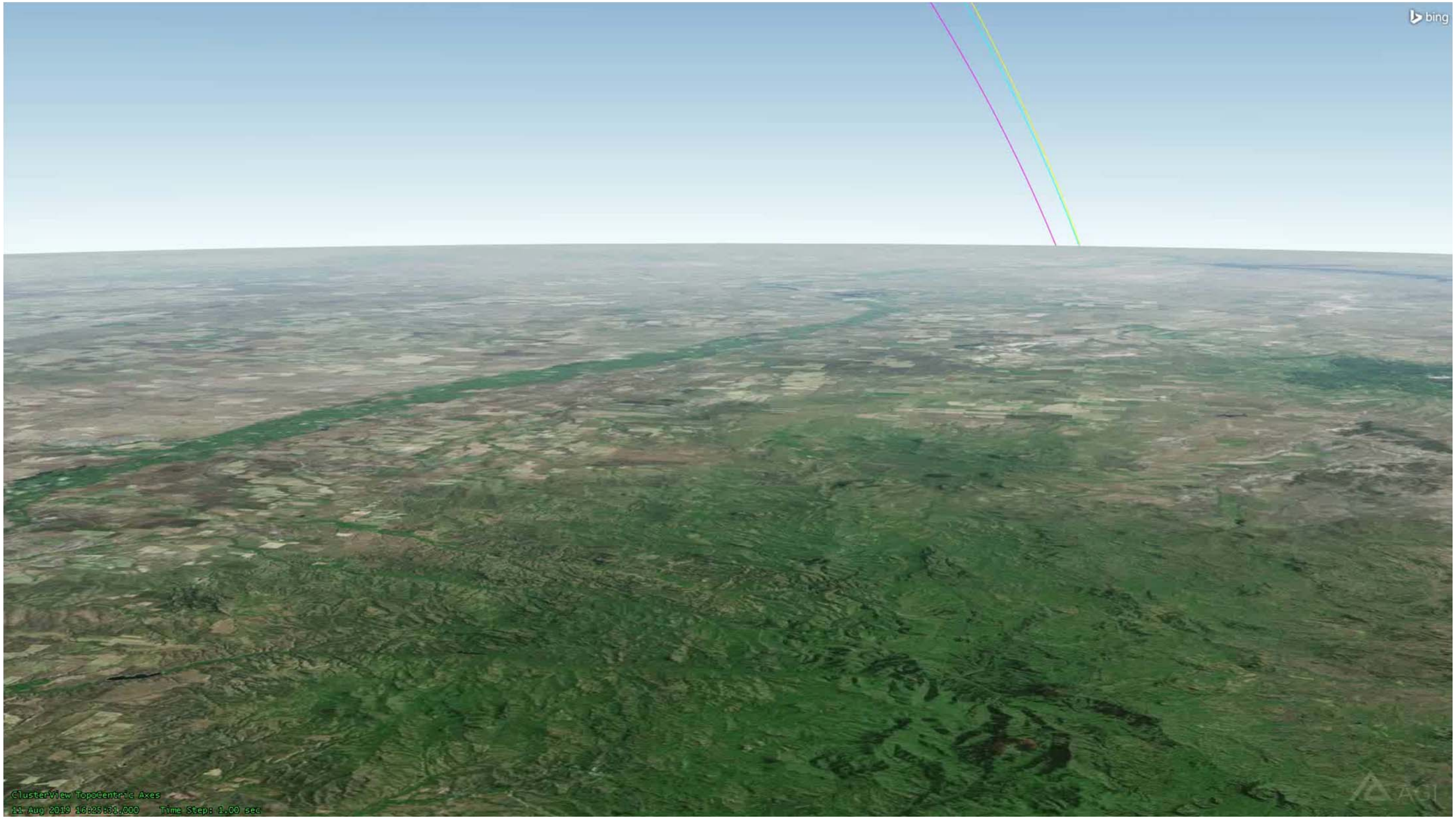
Time Step: 1.00 sec


ClusterView TopoCentric Axes
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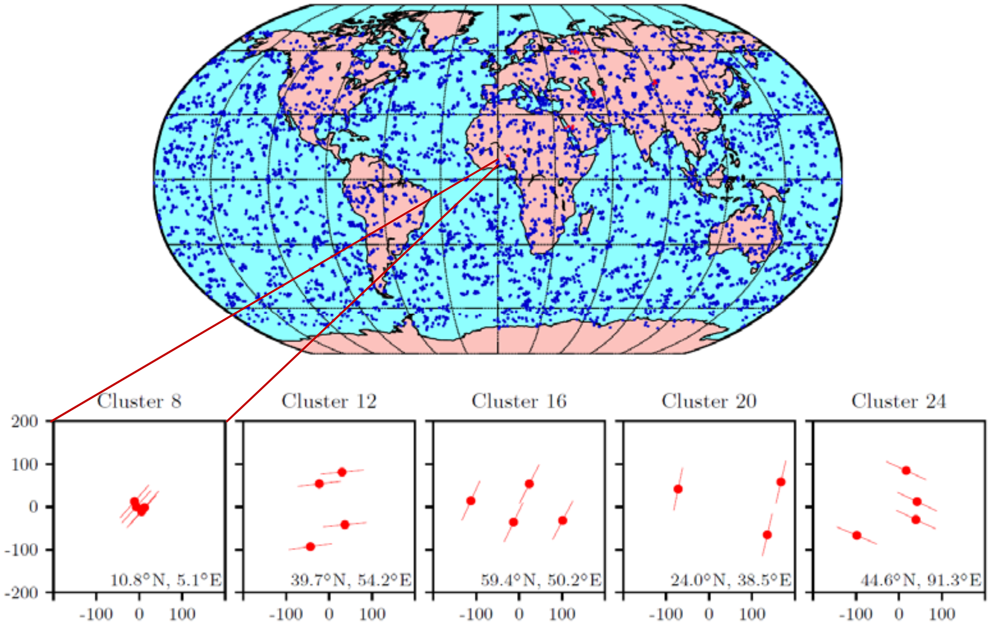
Time Step: 1.00 sec

Cluster Distribution and Quality

Distribution of Occultation Clusters

- MOGs can yield clustered soundings spaced evenly around the world (within latitude constraints).
- This would not be consistently achieved by random or string-of-pearls constellations.



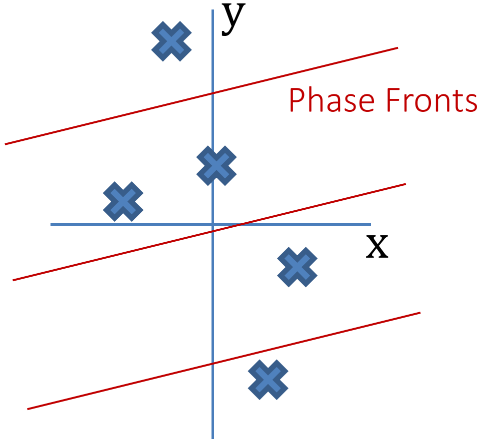
Occultation Cluster Quality Metrics

- χ^2 minimization relates sounding position to uncertainty in wave vector reconstruction.

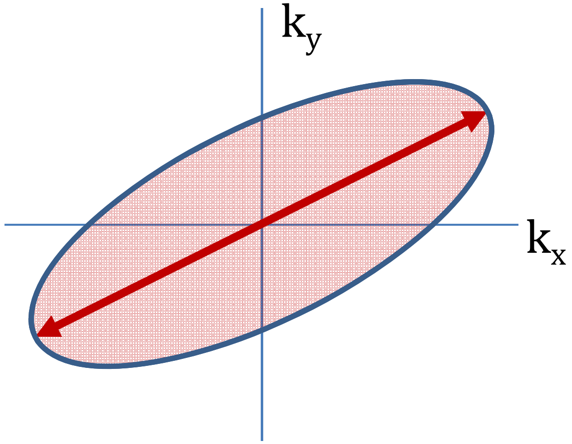
$$\chi^2 = \frac{1}{\sigma_\phi^2} \sum_{i=1}^{n} (\boldsymbol{k} \cdot \boldsymbol{r}_i + \phi_0 - \phi_i)^2$$

- $q_1 \propto$ uncertainty area
- $q_2 \propto$ least-certain axis

Occultation Cluster



Wave Vector Uncertainty

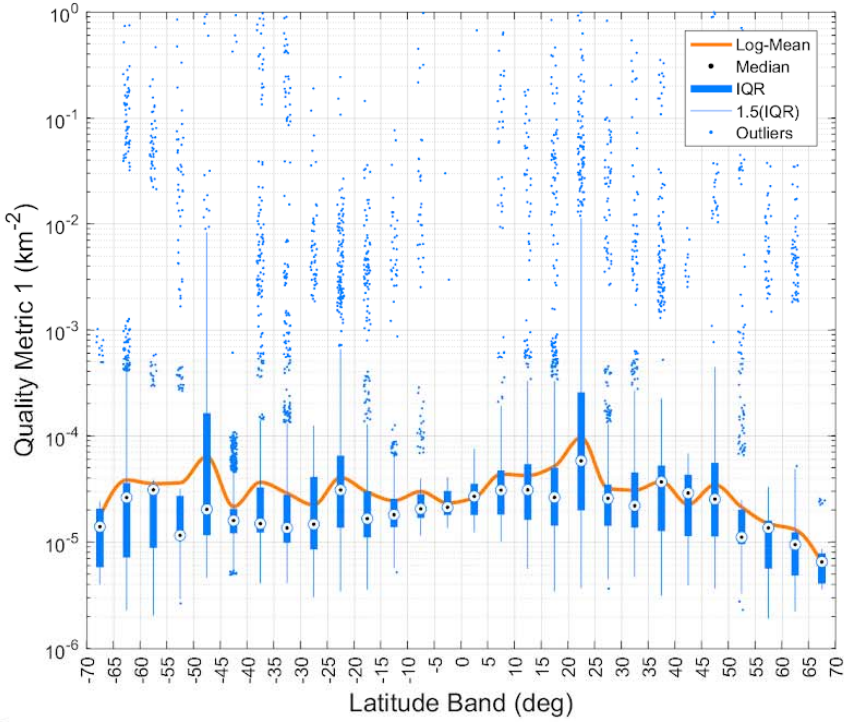


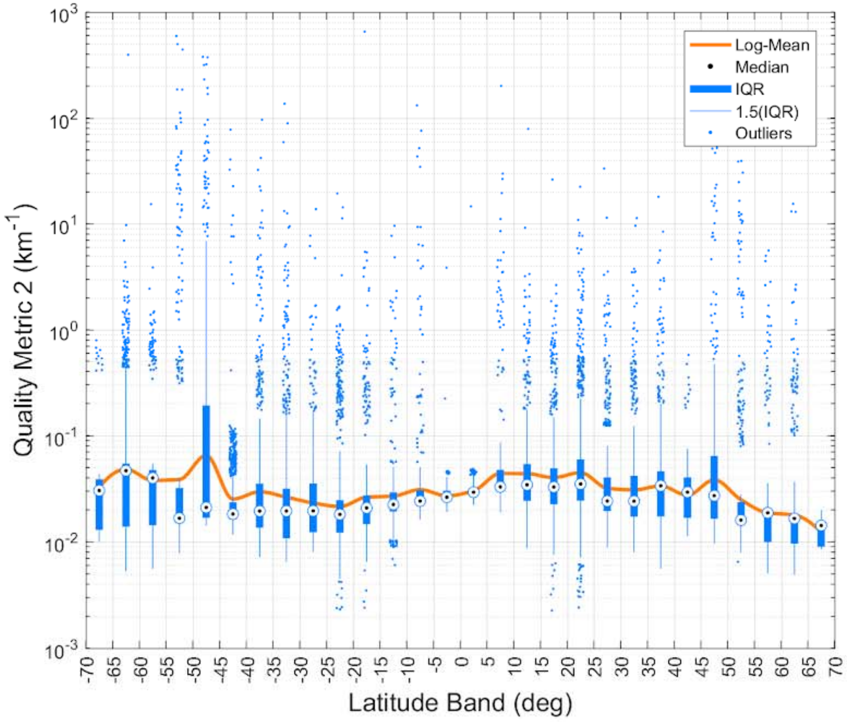
$$R = \begin{bmatrix} \delta x_1 & \delta y_1 \\ \vdots & \vdots \\ \delta x_n & \delta y_n \end{bmatrix}$$

$$q_1 = \sigma_\phi^2 \left(\det R^\mathsf{T} R\right)^{-1/2}$$

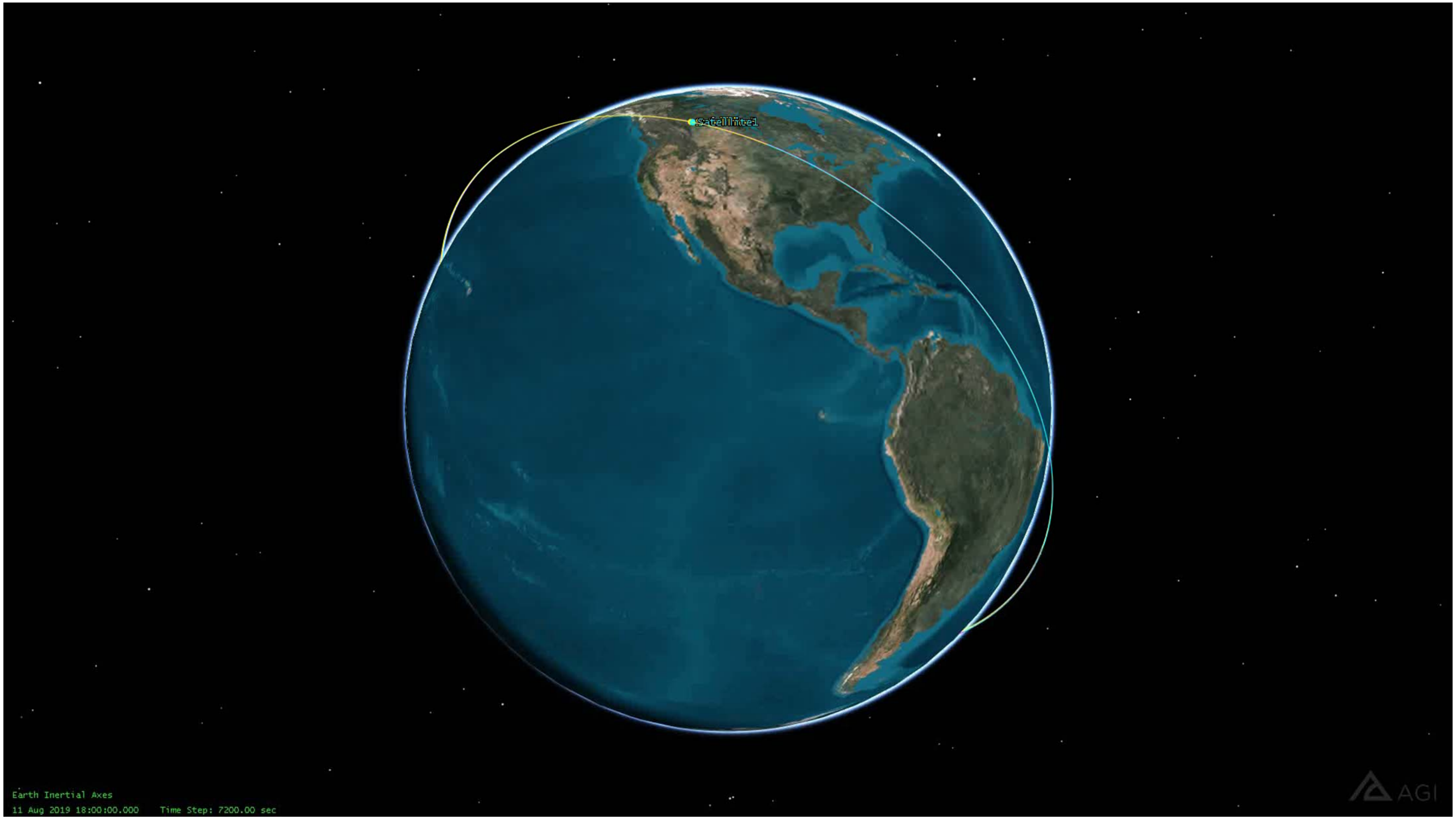
$$q_2 = \sigma_\phi \left[\lambda_{\min}\left(R^\mathsf{T} R\right)\right]^{-1/2}$$

Quality Metrics ($\sigma_\phi = 1$) v. Latitude





Constellation Maintenance


Satellite1
Earth Inertial Axes
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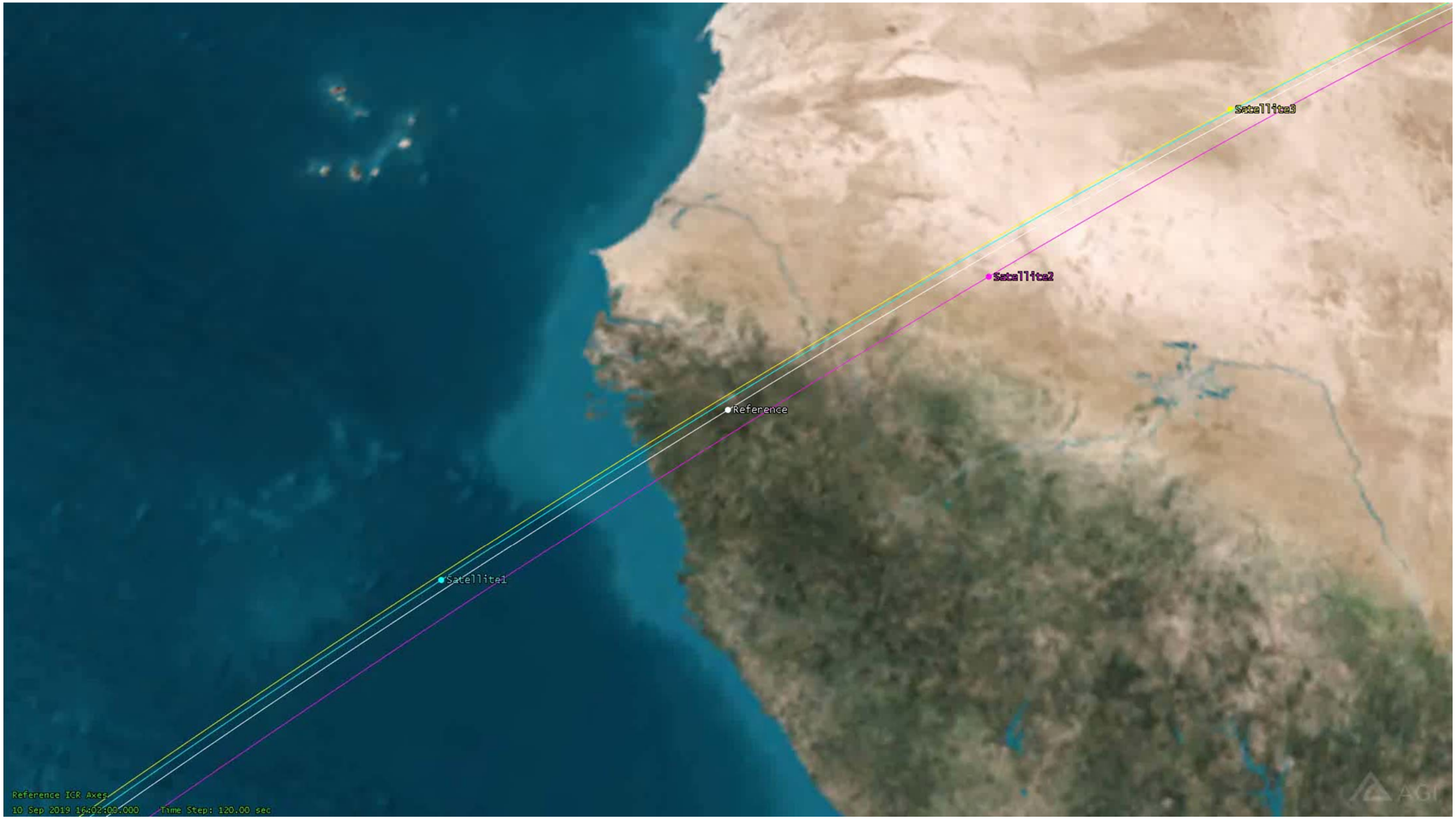
AGI


Satellite3
Satellite2
Reference
Satellite1
Reference ICR Axes
10 Sep 2019 16:02:00.000
Time Step: 120.00 sec
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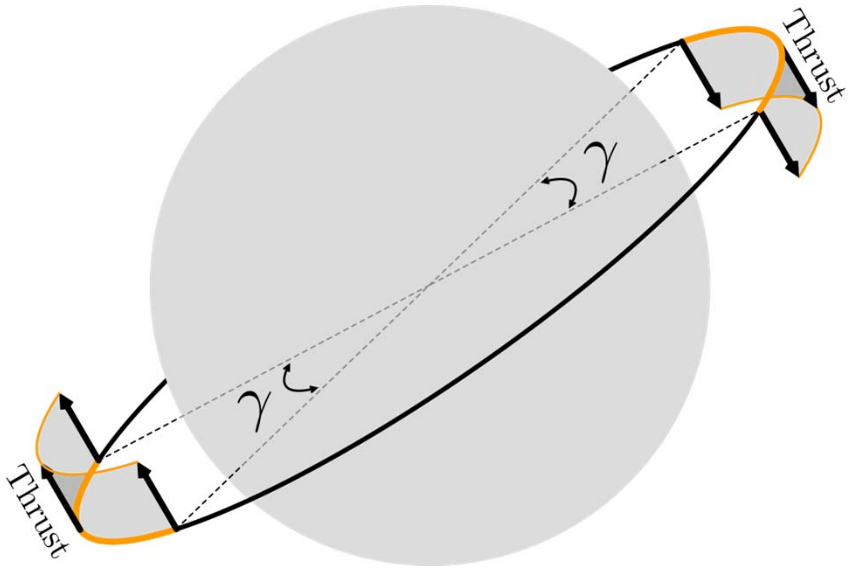
Out-of-Plane Thrusts

- Symmetric arcs of thrust at the maximum and minimum latitudes can counter J_2 torques.
- Angle γ gives required thrust arc length:
  - Impulsive propulsion lets γ approach zero.
  - Electric propulsion requires that γ be a substantial fraction of the orbit arc.



Burn Arc Length and Efficiency

- Rate of ΔV use is determined by ideal rate (α^*, from orbit geometry) and efficiency (η, from burn arc):

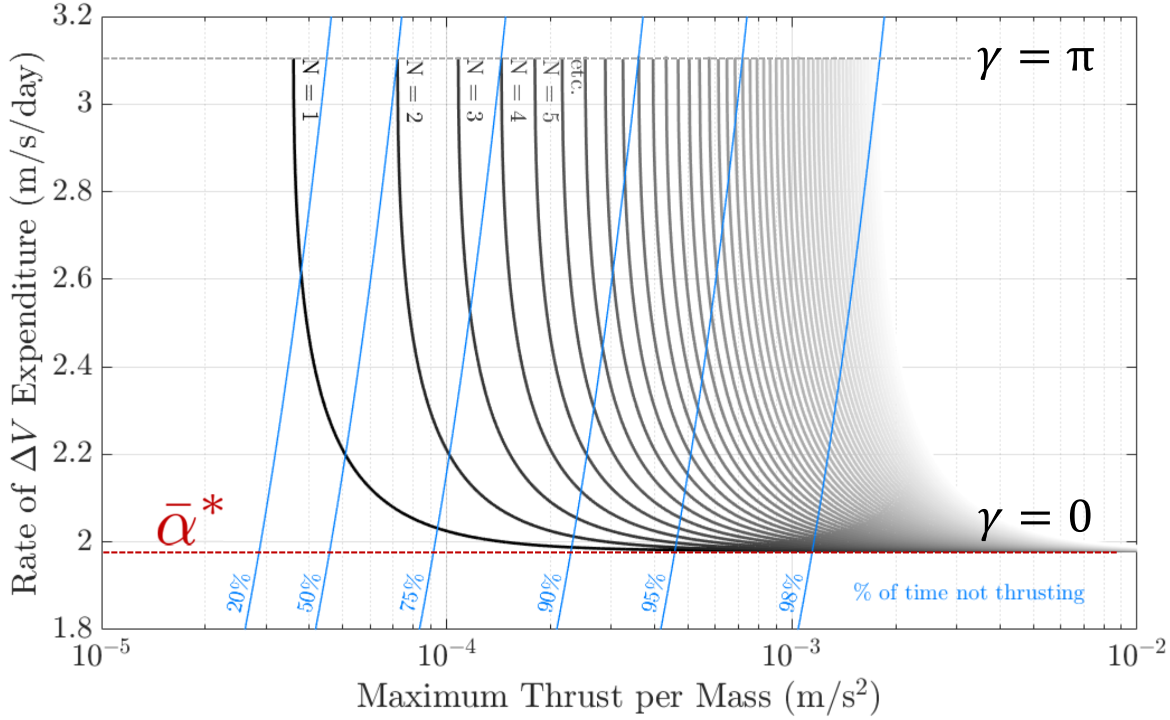
$$\bar{\alpha}_j \equiv \bar{\alpha}_j^*/\eta \qquad \eta = \operatorname{sinc} \gamma/2$$

- For correction every N^{th} orbit with thrust/mass α_{max} :

$$\bar{\alpha}_j = \frac{2\alpha_{\max}}{N\pi} \sin^{-1}\left(\frac{N\pi}{2} \frac{\bar{\alpha}_j^*}{\alpha_{\max}}\right)$$

Rate of ΔV Expenditure

- Example (ISS):
  - $i = 51.4^{o}$
  - $\delta = 0.172^{o}$
  - $h = 400\ km$
- Frequent burns or higher thrust per mass raises efficiency.



Maintenance & SmallSat Propulsion

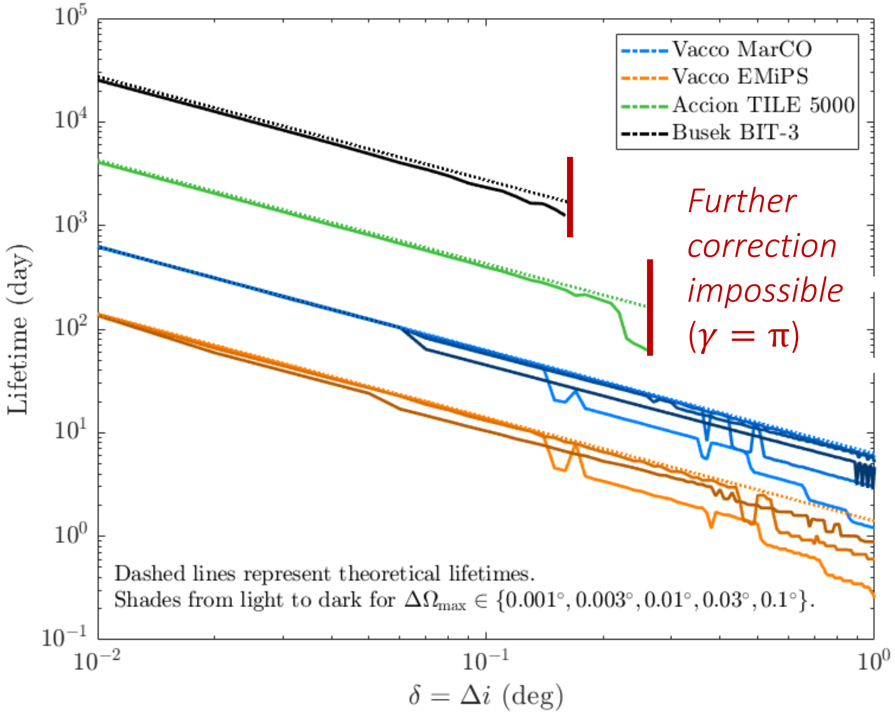
- Trade study is necessary to choose propulsion option:
- High-thrust options (e.g. cold gas) able to correct for larger spacecraft separations. They have small γ and high η.
- Low-thrust options (e.g. electrospray) have appreciably higher total ΔV capacity, and therefore longer lifetime despite less efficient operation.

Vacco JPL MarCO
(Cold Gas Thruster)

Accion TILE 5000
(Ionic Liquid Electrospray)

Constellation Maintenance Lifetime

- Simulations performed for 10 kg small satellite, varying propulsion option.
- Rate of *ΔV* expenditure is driven by inclination separation of satellites.
- Increased specific impulse of electric propulsion provides greatly increased lifetime *if correction is possible.*



Further correction impossible ($\gamma = \pi$)

Dashed lines represent theoretical lifetimes.
Shades from light to dark for $\Delta\Omega_{\text{max}} \in \{0.001^\circ, 0.003^\circ, 0.01^\circ, 0.03^\circ, 0.1^\circ\}$.

Conclusions

- Formation-flying satellites can be used to yield frequent occultation clusters for tomography of internal gravity waves.
- Long burn arcs with electric propulsion necessary for long lifetimes, but mutual orbit group separation is limited by available thrust/mass of available systems.
- Ability to perform precision orbit determination during low-thrust burn arcs drives occultation yield of the constellation if long lifetimes are desired.

Questions?

Thank you to the National Science Foundation for funding this study.