# A bi-local estimation approach for residual ionospheric correction of radio occultation bending angles


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**Abstract**

We have developed the theory for a bi-local estimation of residual ionospheric errors in bending angle profiles retrieved from radio occultation (RO) measurements. Bi-local in this context refers to the situation where the electron density is different, though still spherically stratified, on the transmitter-inbound and receiver-outbound sides of the RO tangent points. As opposed to local spherical symmetry, we call this bi-local spherical symmetry. So far, theoretical estimates of ionospheric residual errors have been based on the assumption of local spherical symmetry. We here extend such estimates to the case of bi-local spherical symmetry. The theory also takes into account the contribution from the geomagnetic field in the ionospheric refractive index, and as well allows for a non-zero local electron density at the receiver in orbit. As part of the derivations, we found a small term not previously noted, which can become appreciable for elliptical satellite orbits. The results were verified by ray tracing through simple models of the ionospheric electron density and geomagnetic field. The accuracy of a residual error correction based on these results would be limited by the uncertainty in knowledge of the ionospheric electron density and by horizontal electron density gradients along the ray paths. Finally we point to results from follow-on work that applied the theory to test-day ensembles of real RO data from Metop, GRACE, and CHAMP.


## Models and setup

Results are based on series expansions to order $f^{-4}$, including terms of possible size $\sim 10^{-2}\,\mu$rad with the help of ray tracing and retrieval simulations using non-circular LEO orbit at $\sim$450 km, spherically symmetrical geomagnetic field, and model profiles as shown in **Figure 1**. Various parameters are defined in **Figure 2**.

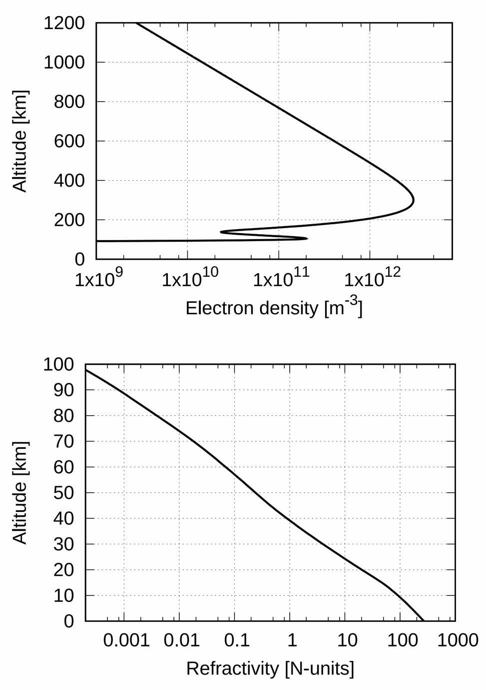
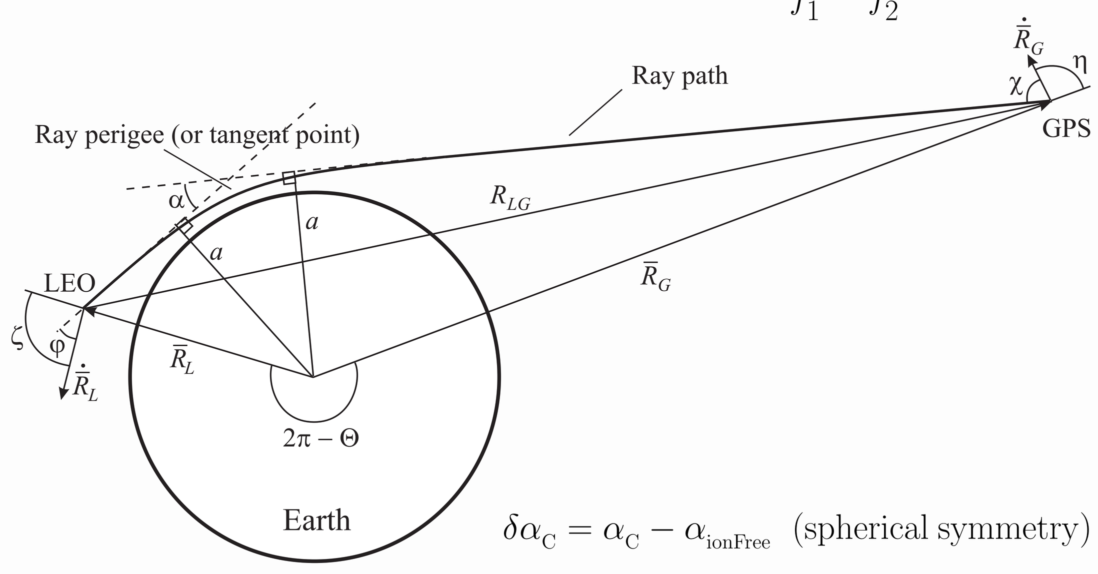



**Figure 1:** Model profiles used in ray tracing. Top: Ionospheric electron density. Bottom: Atmospheric refractivity.

$$\text{standard ionospheric correction :}\quad \alpha_{\rm C} = \frac{f_1^2\alpha_1(a) - f_2^2\alpha_2(a)}{f_1^2 - f_2^2}$$




$$\delta\alpha_{\rm C} = \alpha_{\rm C} - \alpha_{\rm ionFree}\quad \text{(spherical symmetry)}$$

**Figure 2:** The occultation geometry, defining various parameters. $\dot{\bar{R}}_L$ and $\dot{\bar{R}}_G$ are the projections of the satellite velocities into the occultation plane (adapted from (Melbourne et al., 1994)). For the equations below we define $v_{\rm L} = |\dot{\bar{R}}_{\rm L}|$, $v_{\rm G} = |\dot{\bar{R}}_{\rm G}|$, $r_{\rm L} = |\bar{R}_{\rm L}|$, and $r_{\rm G} = |\bar{R}_{\rm G}|$.

## Errors in the standard ionospheric correction

$$\delta\alpha_{\rm C} = -\frac{K\mathcal{F}(B_{\|}N_{\rm e})}{f_1 f_2(f_1+f_2)} - \frac{1}{2}\frac{C^2}{f_1^2 f_2^2}\frac{1}{a}\frac{{\rm d}[a^2\mathcal{F}(N_{\rm e}^2)]}{{\rm d}a} - \frac{C}{(f_1^2-f_2^2)}\frac{r_{\rm L}H_{\rm L}N_{\rm e}(r_{\rm L})}{\sqrt{r_{\rm L}^2-a^2}}\frac{{\rm d}(\alpha_1-\alpha_2)}{{\rm d}a}.$$

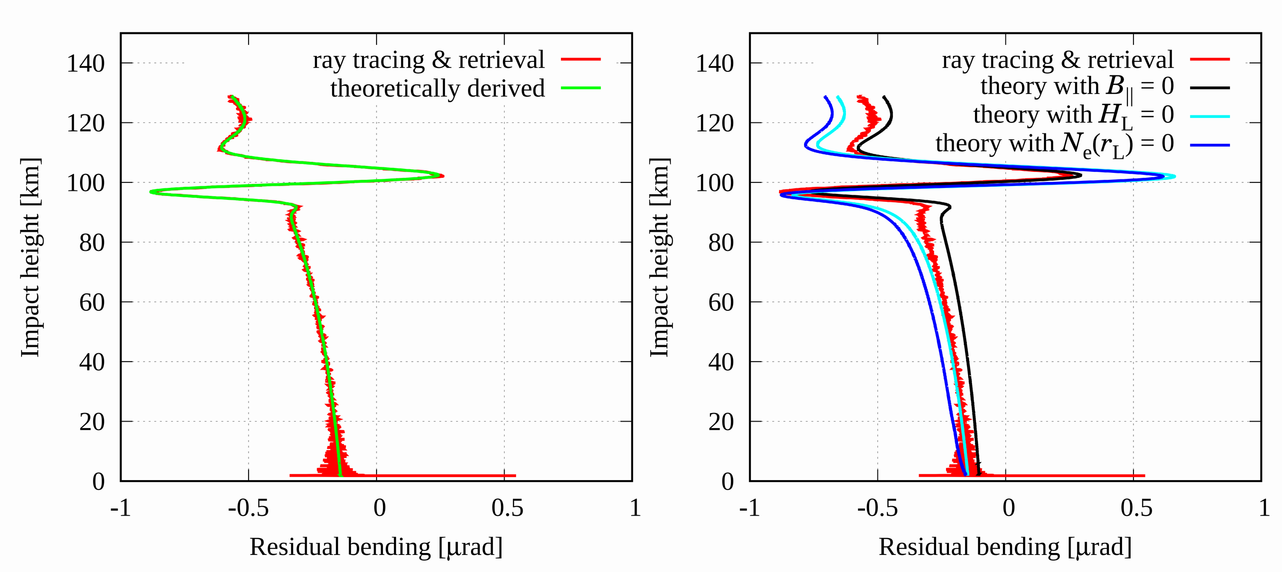



**Figure 3:** Residual bending angle errors in the standard ionospheric correction as a function of impact height. Left: Simulations compared to theory according to the full expression above. Right: Simulations compared to theory leaving out certain terms.

## Errors due to non-circular orbits

$$\Delta a = \frac{C}{f^2}\frac{r_{\rm L}H_{\rm L}N_{\rm e}(r_{\rm L})}{\sqrt{r_{\rm L}^2-a^2}}, \qquad H_{\rm L} \approx \frac{v_{\rm L}\cos\zeta}{(v_{\rm G}r_{\rm G}^{-1}\sin\eta - v_{\rm L}r_{\rm L}^{-1}\sin\zeta)}.$$

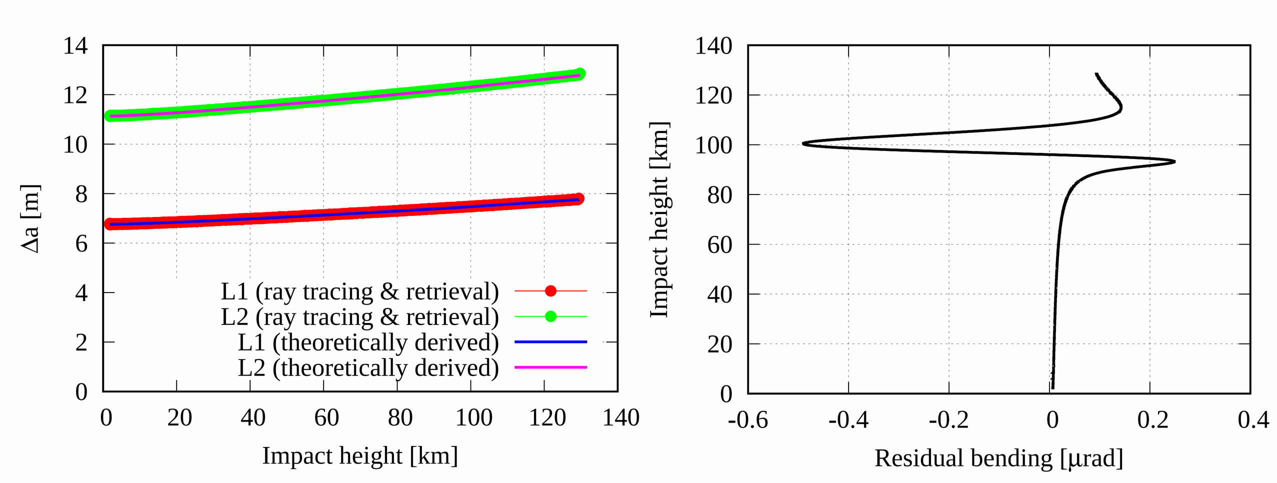



**Figure 4:** Errors resulting from ignoring the electron density at the LEO satellite for a simulated case with a non-circular LEO orbit. Left: The error in the derived impact parameter. Right: The residual error in the ionospheric corrected bending angle.

## Relation to previous theoretical works

In terms of Vorob'ev and Krasil'nikova (1994):

$$\delta\alpha_{\rm C}^{[B_{\|}=0,N_{\rm e}(r_{\rm L})=0,N_{\rm e}(b)=0]} = -\frac{C^2}{f_1^2 f_2^2}a\int_b^\infty \frac{{\rm d}N_{\rm e}^2}{{\rm d}x}\frac{(3x^2-2a^2){\rm d}x}{(x^2-a^2)^{3/2}}.$$

In terms of Syndergaard (2000):

$$\delta\alpha_{\rm C}^{[B_{\|}=0,H_{\rm L}=0]} = -\frac{1}{2}\frac{C^2}{f_1^2 f_2^2}a\frac{{\rm d}^2}{{\rm d}a^2}\int_{\rm F} N_{\rm e}^2{\rm d}s - \frac{C^2}{f_1^2 f_2^2}\frac{\rm d}{{\rm d}a}\int_{\rm F} N_{\rm e}^2{\rm d}s.$$

In terms of Healy and Culverwell (2015):

$$\delta\alpha_{\rm C}^{[B_{\|}=0,N_{\rm e}(r_{\rm L})=0,N_{\rm e}(b)=0]} \approx -\frac{3}{8\pi}\frac{f_1^2 f_2^2}{(f_1^2-f_2^2)^2}\frac{r_{\rm m}\sqrt{r_{\rm m}^2-a^2}}{aH}[\alpha_1(a)-\alpha_2(a)]^2.$$

## Bi-local spherical symmetry

$$\text{Bouguer's law :}\quad a = r\sin\psi = \text{constant}$$

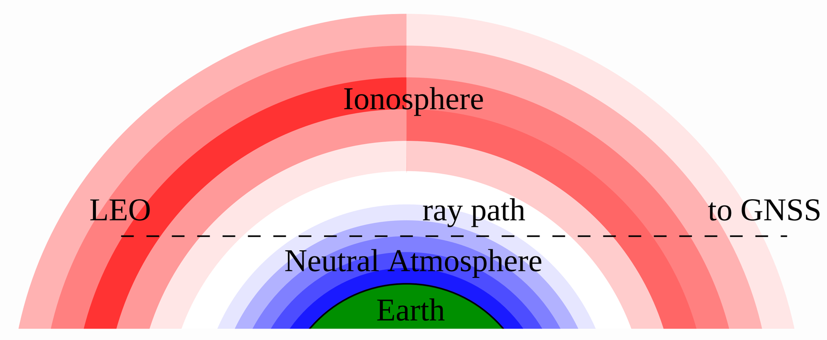



**Figure 5:** Illustration of bi-local spherical symmetry. The neutral atmosphere is spherically symmetrical, and the ionosphere is spherically stratified on each side of the tangent points such that Bouguer's law can be applied for rays with tangent points below the ionosphere.

- Assume that the ionosphere is different on each side of the tangent points, but still spherically stratified away from the tangent points (bi-local symmetry)
- Then the impact parameter for ray paths with tangent points well below the ionosphere (below 80 km where it is relevant for neutral atmosphere retrieval purposes) is still invariant along the path
- Derivations leading to the residual errors in the standard correction are still valid in the case of bi-local spherical symmetry for impact parameters with tangent points below the ionosphere, we just need to emphasize the possiblity that the ionosphere on the GNSS side can be different from the ionosphere on the LEO side:

$$\mathcal{F}(X) = \int_a^{r_{\rm G}}\frac{{\rm d}X}{{\rm d}x}\bigg|_{\rm G}\frac{a{\rm d}x}{\sqrt{x^2-a^2}} + \int_a^{r_{\rm L}}\frac{{\rm d}X}{{\rm d}x}\bigg|_{\rm L}\frac{a{\rm d}x}{\sqrt{x^2-a^2}} - \frac{aX(r_{\rm L})}{\sqrt{r_{\rm L}^2-a^2}}.$$

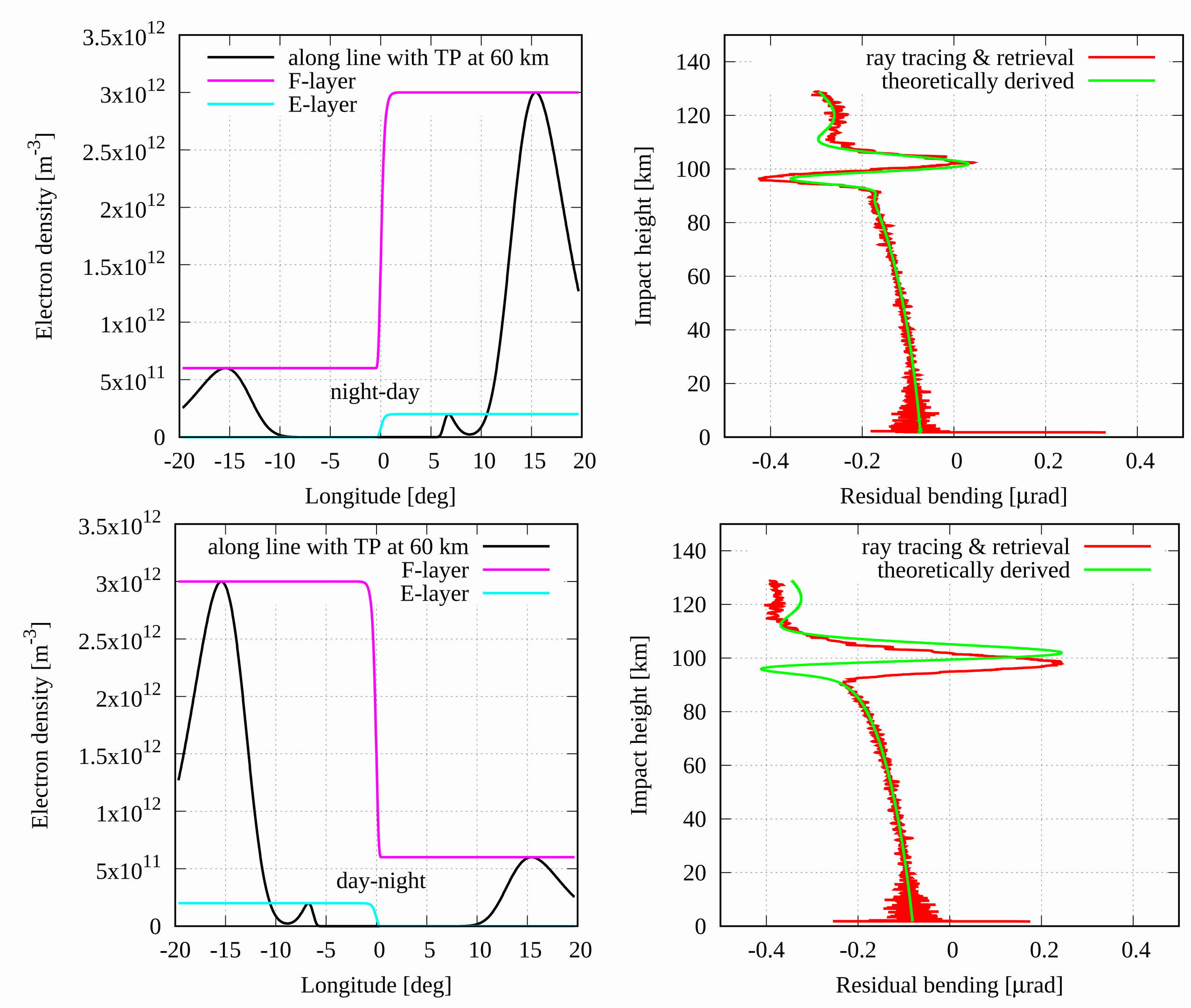



**Figure 6:** The ionospheric horizontal change and the residual bending angle errors in the standard ionospheric correction as a function of impact height in two cases of bi-local spherical symmetry. Top: Night-day transition. Bottom: Day-night transition.

## Limitations due to horizontal gradients

The theoretically derived residual errors, whether based on the results here or using the results by, e.g., Healy and Culverwell (2015), can be significantly off when there are horizontal gradients along the ray paths.

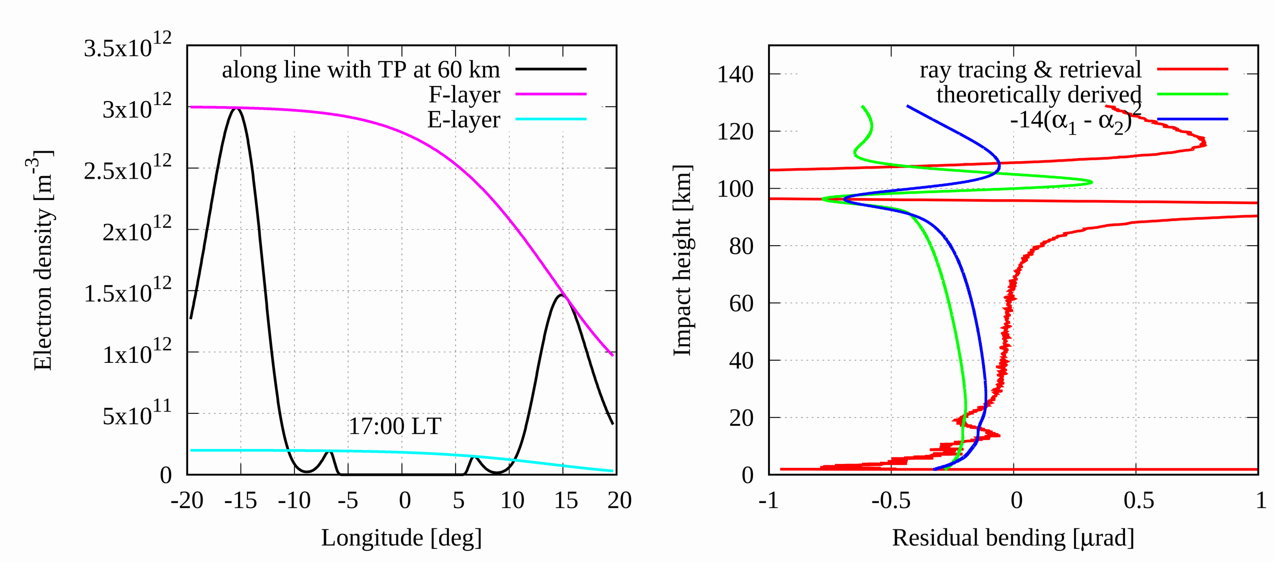



**Figure 7:** The ionospheric horizontal change and the residual bending angle errors in the standard ionospheric correction as a function of impact height in a case of significant horizontal gradients along the ray paths.

## Follow-on work and prospects

The bi-local correction method has been applied in follow-on work to test-day ensembles of real Metop, GRACE, and CHAMP data, intercomparing to the standard correction and to the kappa-correction by Healy and Culverwell (2015). The results so far show a clear added value of applying the bi-local method (Liu et al., 2019).

## References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