

## Introduction

The diurnal cycle is relevant to climate because

- (1) It serves as a test of climate models in the troposphere because of the timing of deep convection,
- (2) It serves as a test of whole atmosphere models in the upper stratosphere because of vertical propagation of the solar tides, and
- (3) It can be used to diagnose how under-sampling of the diurnal cycle biases climate data records.

Bayesian interpolation works by fitting data without over-fitting the data. There are several reasons to apply a Bayesian strategy to analyzing the diurnal cycle:

- (1) Radio occultation (RO) data is globally but highly irregularly distributed with unknown implicit spatial and temporal resolution,
- (2) It can explicitly model the diurnal (and semi-diurnal) cycles analytically, and
- (3) It produces a complete error analysis useful for significance testing.

## Bayesian Interpolation

Basis functions are spherical harmonics and sinusoids in diurnal cycle: longitude  $\lambda$ , latitude  $\theta$ , and diurnal time  $\tau$ . Diurnal time can be alternatively defined as local solar time or as UTC time, depending on final objective.

$$\phi_{lmn}(\lambda, \theta, \tau) = P_{lm}(\sin \theta) \operatorname{Re} \left[ (a_{lmn} e^{i(m\lambda+n\tau)} + b_{lmn} e^{i(m\lambda-n\tau)}) \right]$$

Solution for most probable coefficients:

$$\begin{aligned} \mathbf{A} &= \beta \phi' \phi + \alpha \mathbf{C} \\ a_{lmn}, b_{lmn} &= \mathbf{A}^{-1} \phi' \mathbf{d} \\ 2\alpha \chi^2 &= n_{\text{coeffs}} - \alpha \operatorname{Trace} \mathbf{A}^{-1} \end{aligned}$$

The regularizer  $\mathbf{C}$ :  $C_{lmn} = c l^\mu (l+1)^\mu n^\nu$

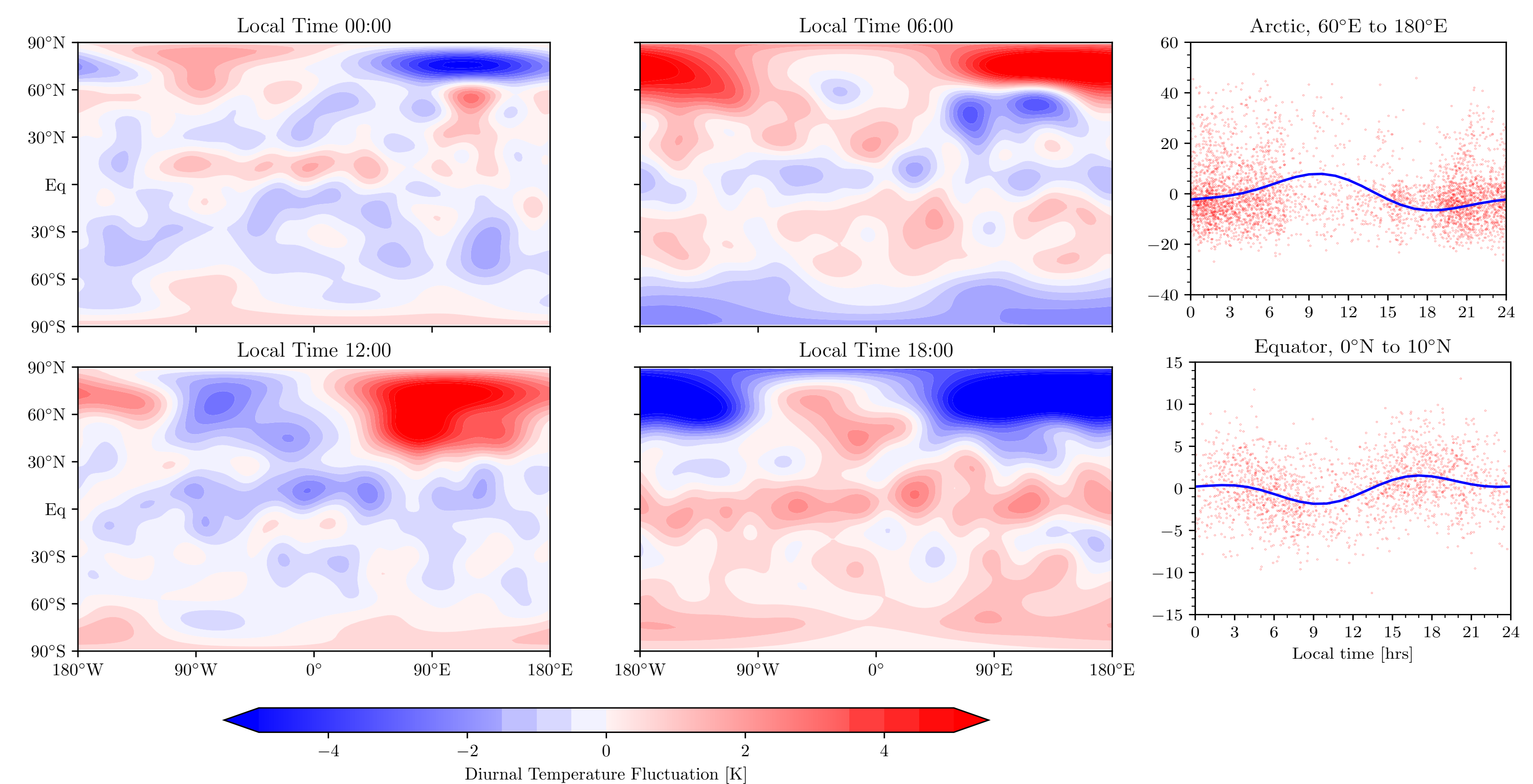
Condition	$c$	$\mu$	$\nu$
$l = m = n = 0$	rMean	0	0
$l > 0, m = n = 0$	rMeridionalGradient	rSpatialExponent	0
$l > 0, m > 0, n = 0$	1	rSpatialExponent	0
$l = m = 0, n > 0$	rHarmonic	0	rHarmonicExponent
$l > 0, n > 0$	rHarmonic	rSpatialExponent	rHarmonicExponent

## References

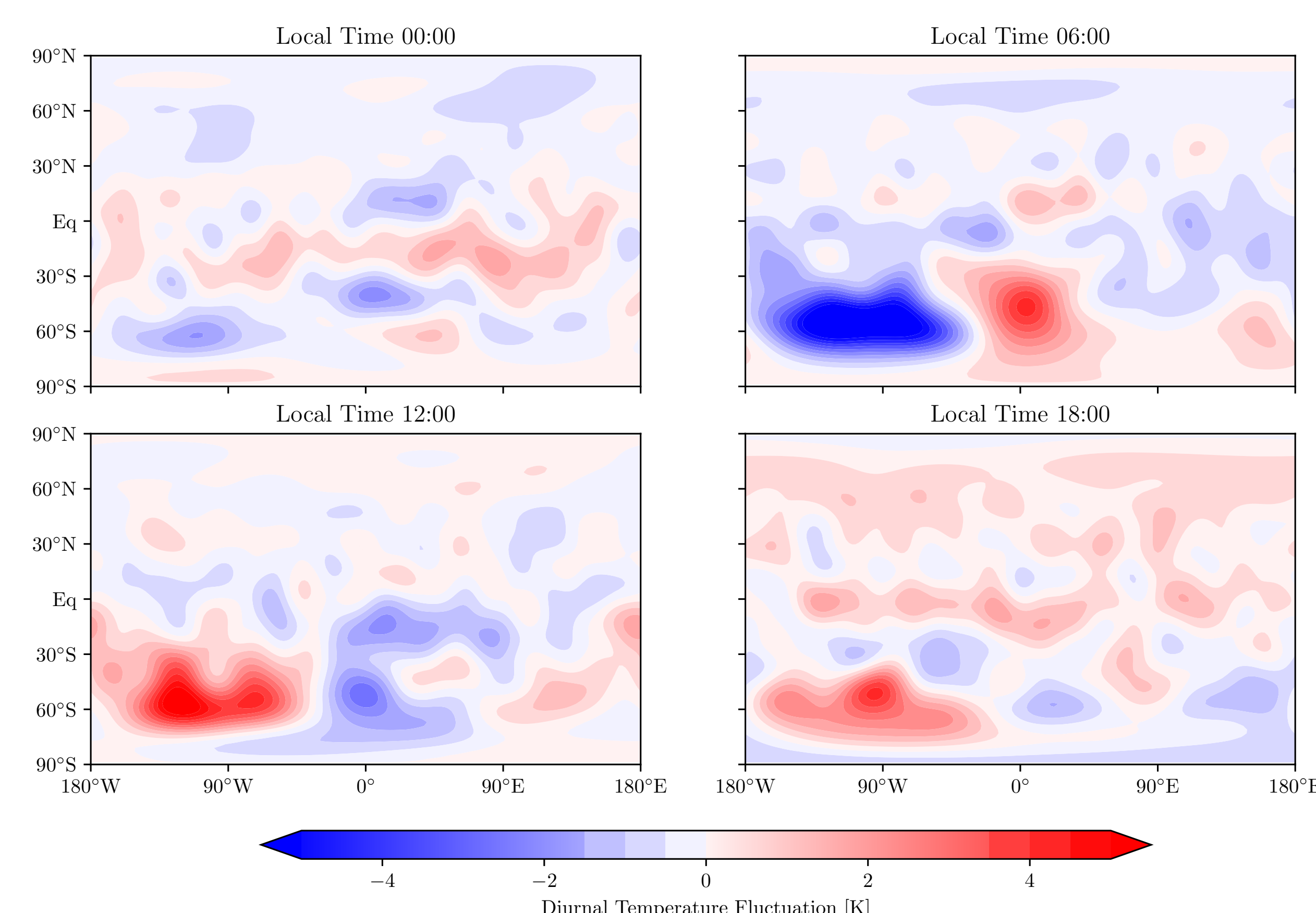
Leroy, S.S., 1997: Measurement of geopotential heights by GPS radio occultation. *J. Geophys. Res.*, **102** (D6), 6971–6986.

Leroy, S.S., C.O. Ao, and O.G. Verkhoglyadova, 2012: Mapping GPS radio occultation data by Bayesian interpolation. *J. Atmos. Ocean. Tech.*, **29**, 1062–1074.

Diurnal cycle in temperature (20 hPa), COSMIC-1, January 2007  
 $\tau$  is local solar time

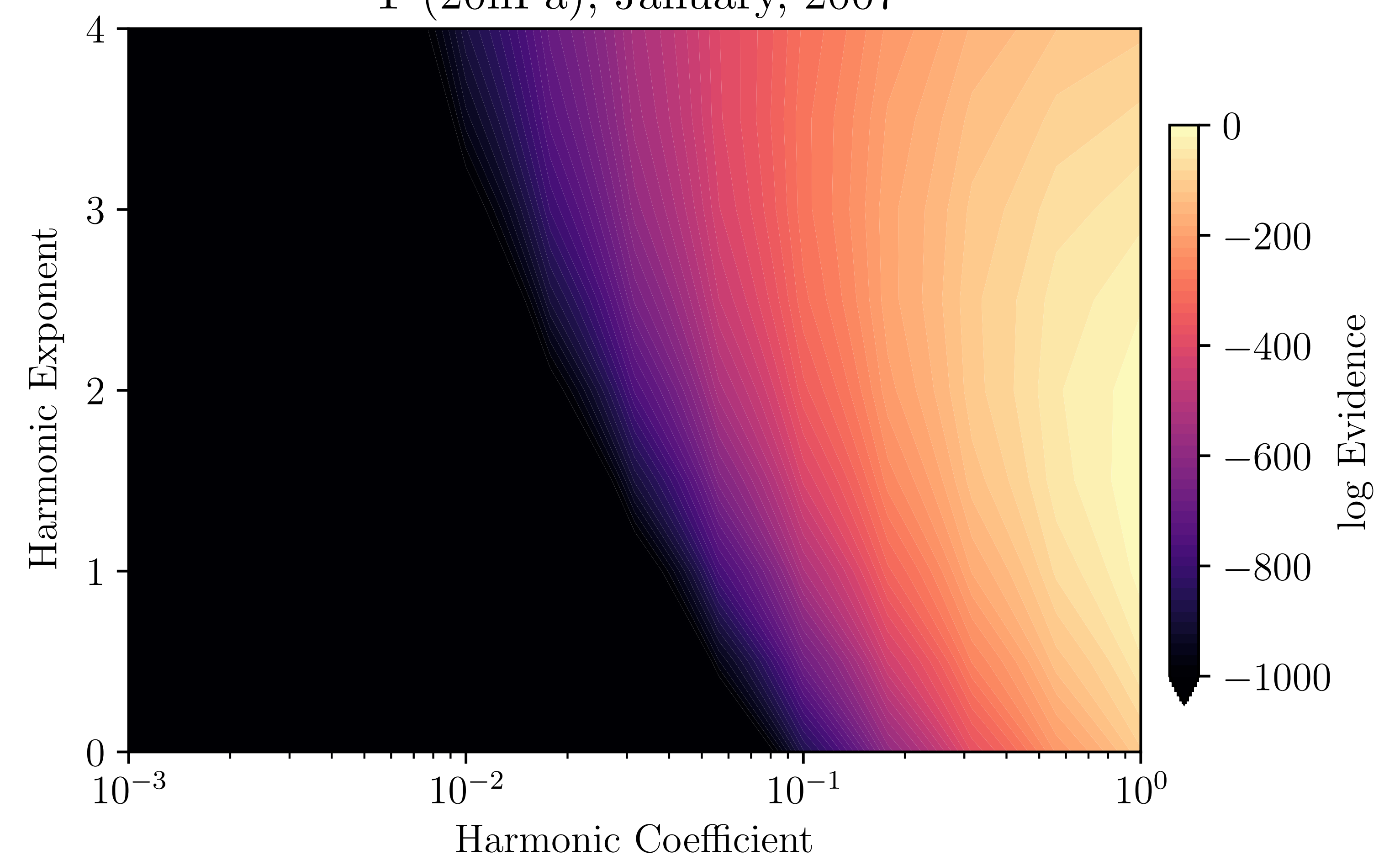


Diurnal cycle in temperature (20 hPa), COSMIC-1, July 2007



## Probability of regularizer parameters

T (20hPa), January, 2007



## Findings

- Bayesian error analysis consistent with bootstrap error analysis (not shown).
- Standard (default) regularizer found objectively.
- *BayesianInterpolation* package available from AER by license agreement.