

Two methods of electron density retrieval from truncated ionospheric radio occultation data

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Outline

1. Background
2. Method 1: SEEIRO
3. Method 2: AVHIRO
4. Assessment
5. Conclusions

1. Background

Goal and context

- ✓ The new EUMETSAT Polar System 2nd Generation (EPS-SG) satellites are designed for neutral atmospheric sounding. The orbit height is in the range **823-848 km**.
- ✓ EPS-SG, will provide as well an opportunity of ionospheric sounding, with impact parameter height only **below 500km**.

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Extrapolation

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Previous work *Extrapolation*

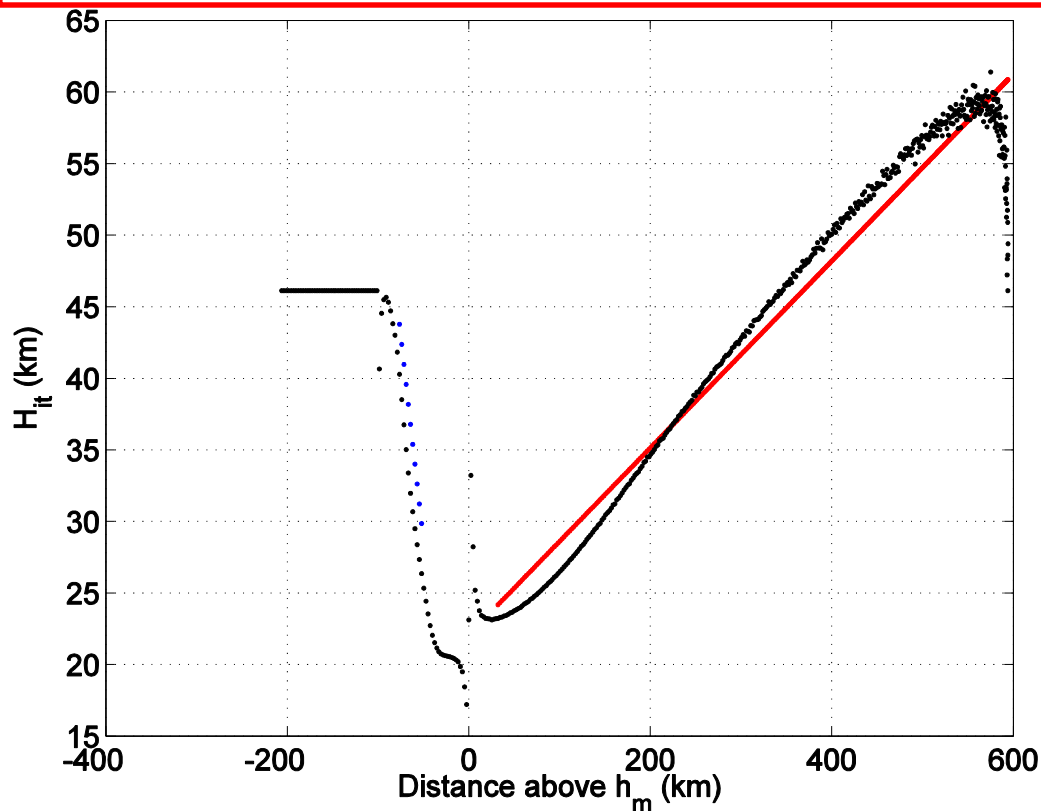
- ✓ The **Vary-Chap model** where the scale height varies linearly with respect to height has been proposed based on FORMOSAT-3/COSMIC GPS occultation data inverted by means of the Improved Abel transform inversion technique, taking into account the horizontal electron content gradients (Olivares-Pulido et al., 2016)
- ✓ A new electron density extrapolation technique - **the Vary-Chapman Extrapolation Technique (VCET)** - for impact parameters of 500km up to the EPS-SG orbit height was developed, when accurate electron density profile below 500km is available (Hernández-Pajares et al., 2017)

The Vary-Chapman Extrapolation Technique (VCET)

$N_m, h_m, H_{0,t}, \partial H_t / \partial h$

$$N = N_m e^{k(1-z-e^{-z})}, \text{ where } z = \frac{h-h_m}{H}$$

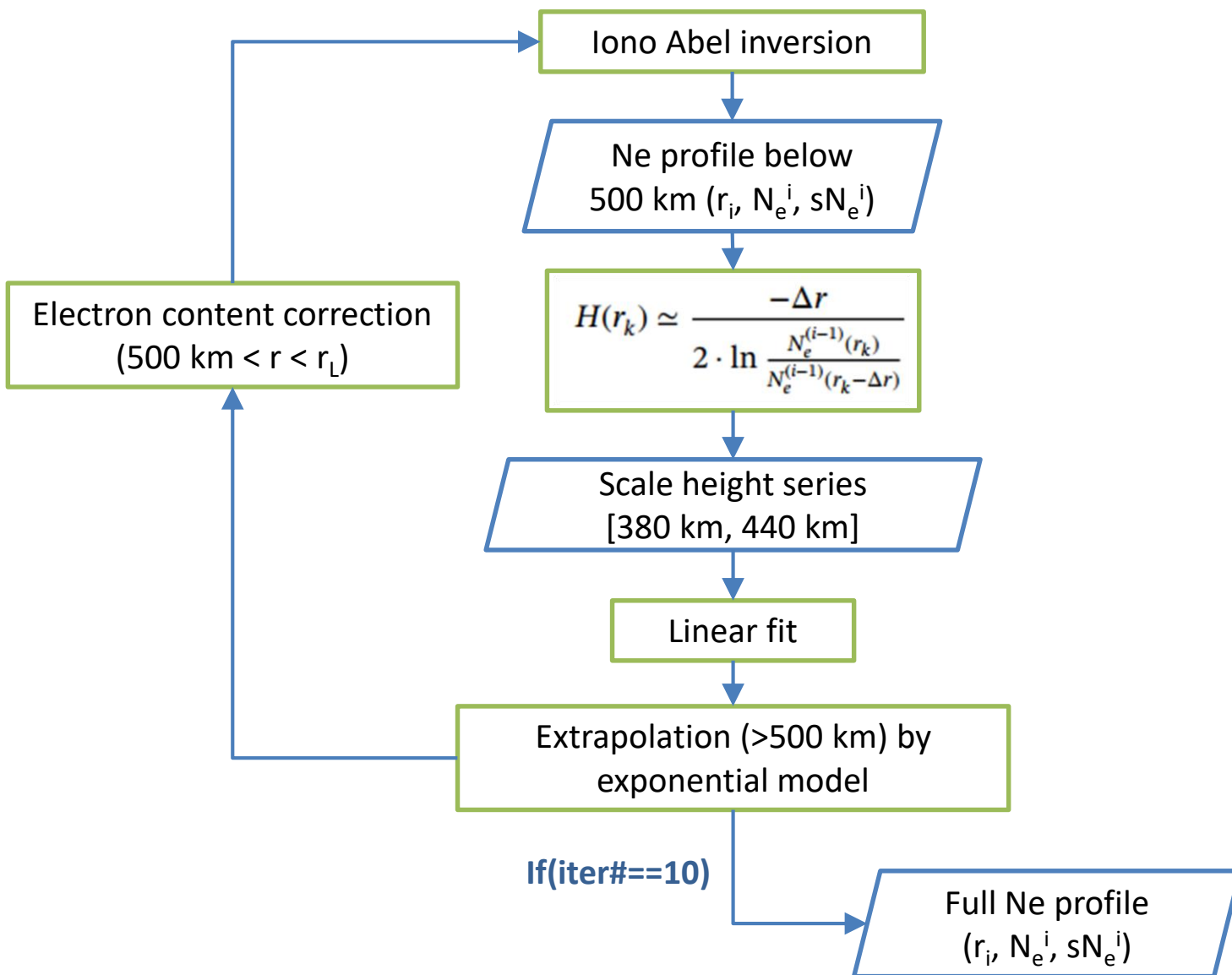
$$H_t(h) = \frac{\partial H_t}{\partial h} (h - h_m) + H_{0,t}, \quad h \geq h_m$$



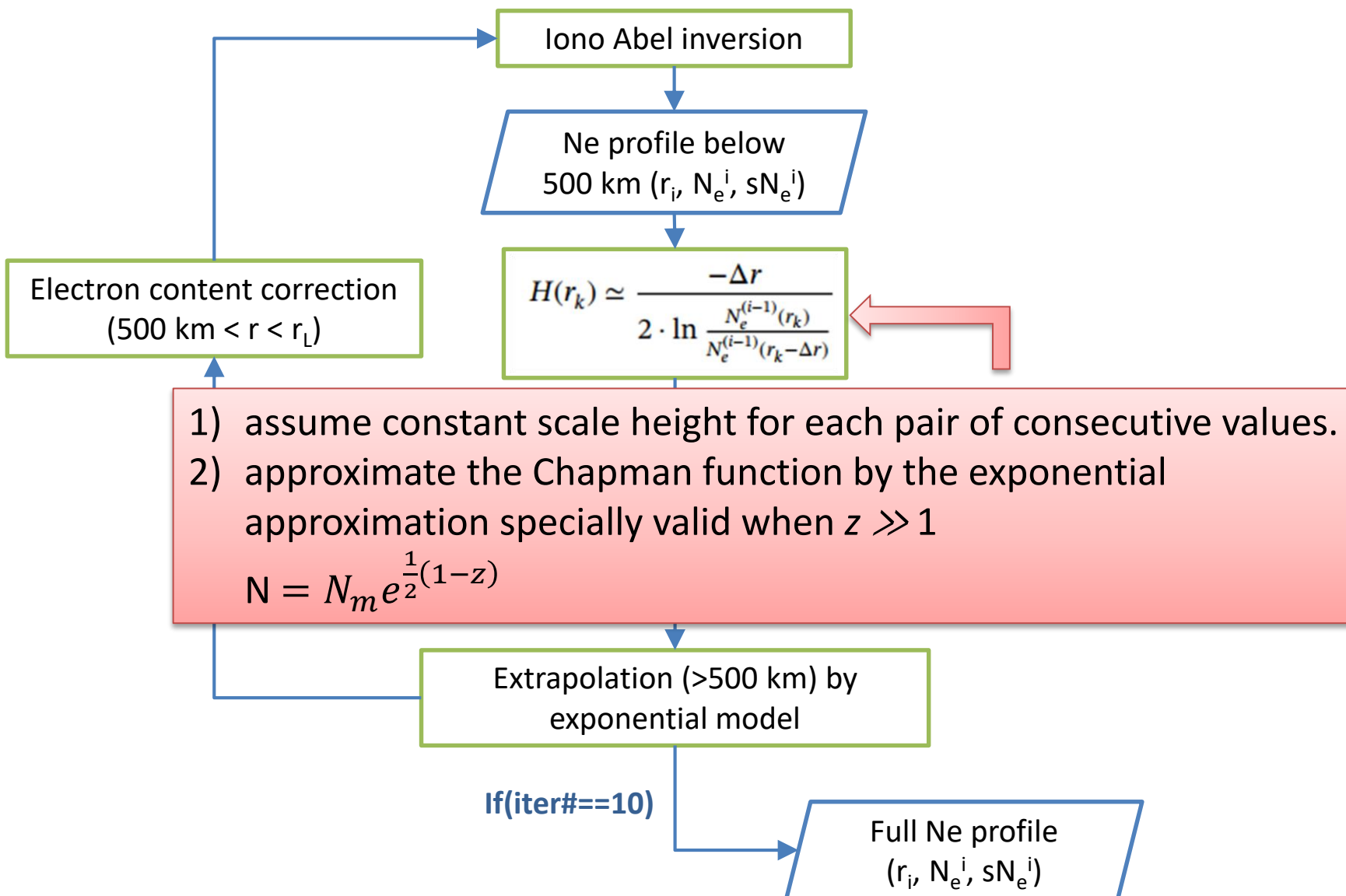
2. Method 1: SEEIRO

the Simple Estimation of Electron density profiles from topside Incomplete RO data

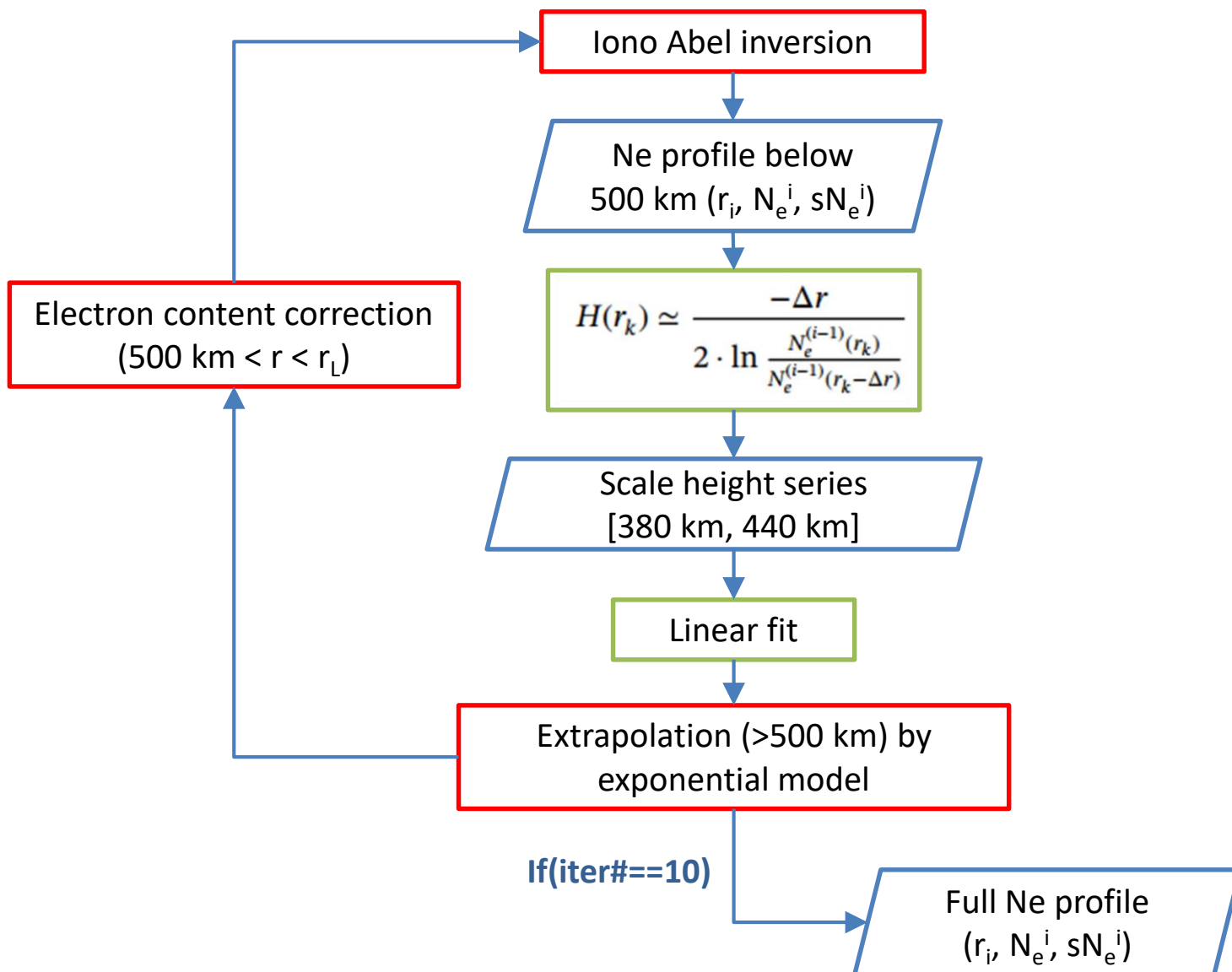
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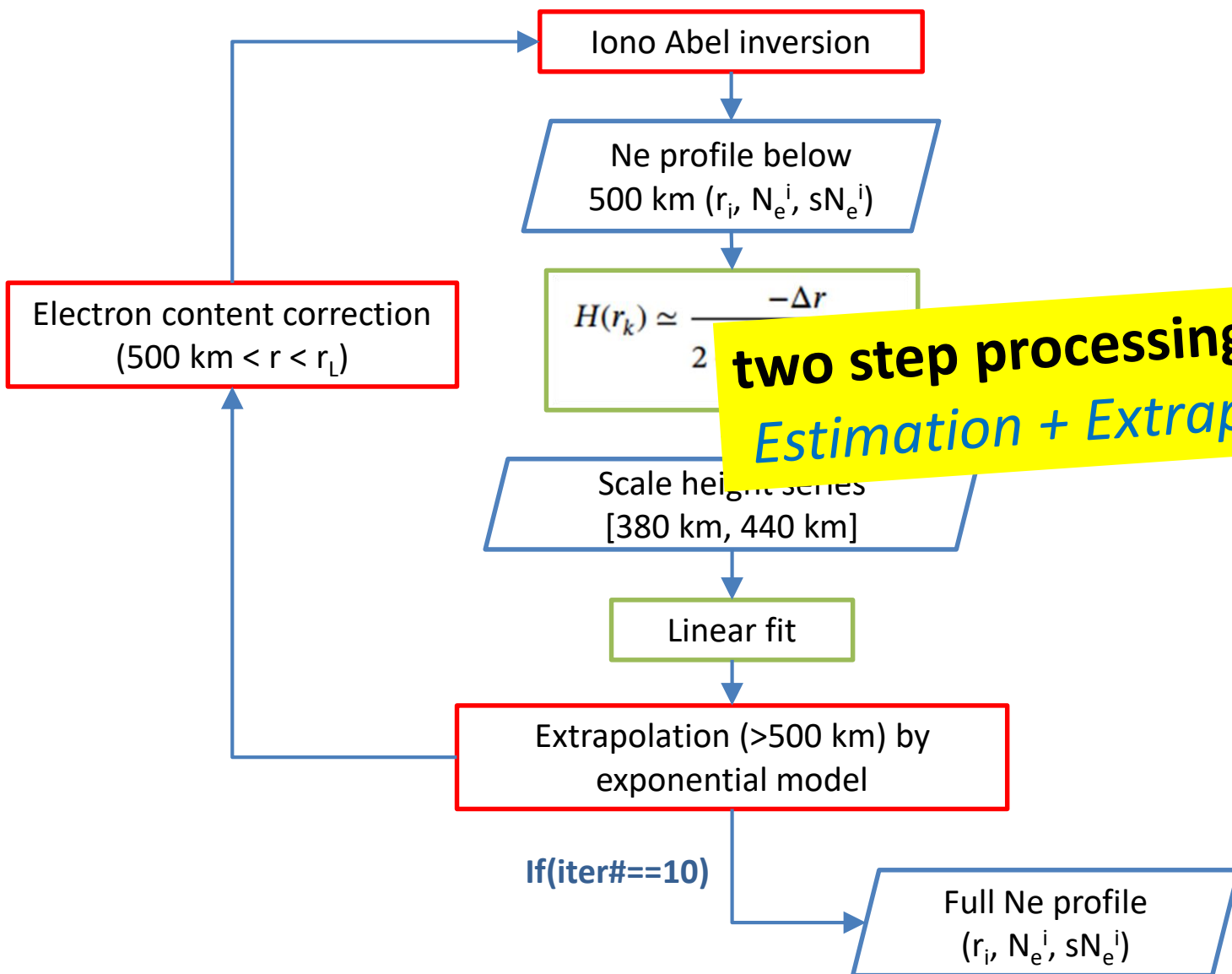
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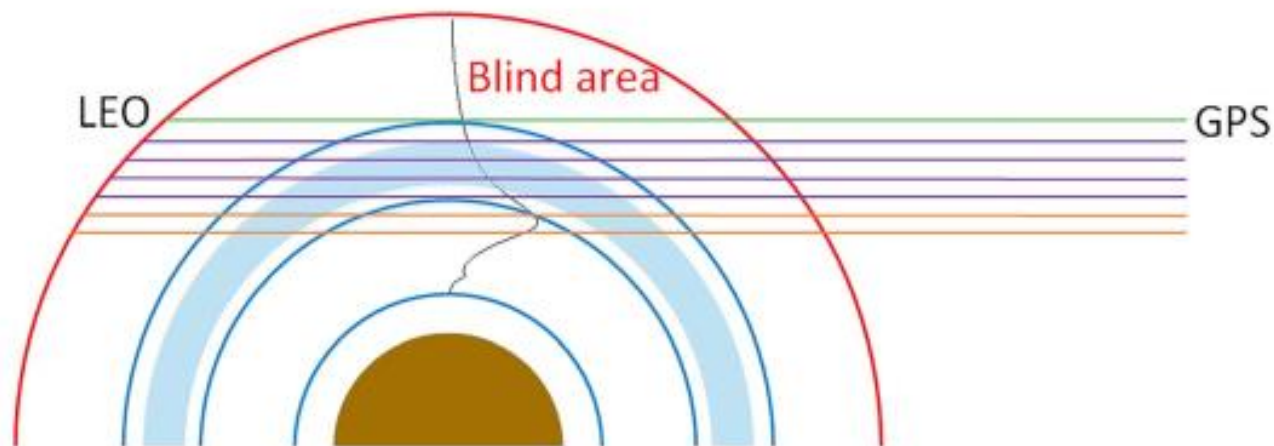


3. Method 2: AVHIRO

the **A**bel-**V**aryChap **H**ybrid modeling
from topside **I**ncomplete **R**O data

Estimate all the unknown parameters simultaneously

3. Method 2: AVHIRO



Observation equations ($Ax=b$): from top to bottom

$$(L_I)_1 = \alpha (2l_{1,1}N_1 + 2l_{1,2}N_2 + \dots + 2l_{1,x}N_x) + B_I$$

$$(L_I)_2 = \alpha (2l_{2,1}N_1 + 2l_{2,2}N_2 + \dots + 2l_{2,x}N_x + 2l_{2,x+1}N_{x+1}) + B_I$$

$$(L_I)_3 = \alpha (2l_{3,1}N_1 + 2l_{3,2}N_2 + \dots + 2l_{3,x}N_x + 2l_{3,x+1}N_{x+1}) + B_I$$

...

$$(L_I)_6 = \alpha (2l_{6,1}N_1 + 2l_{6,2}N_2 + \dots + 2l_{6,x}N_x + 2l_{6,x+1}N_{x+1} + 2l_{6,x+2}N_{x+2}) + B_I$$

....

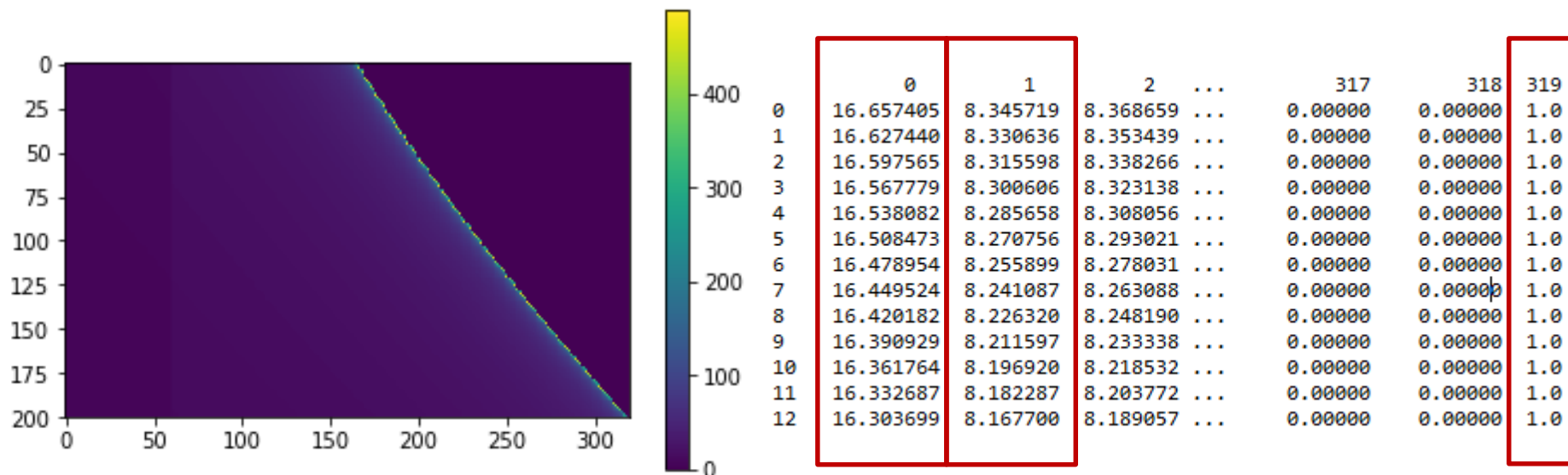
L_i : the ionospheric combination of carrier phases

N_i : electron density in each layer

B_i : the corresponding ambiguity term

3. Method 2: AVHIRO

Example of Design matrix: $A_{200 \times 319}$

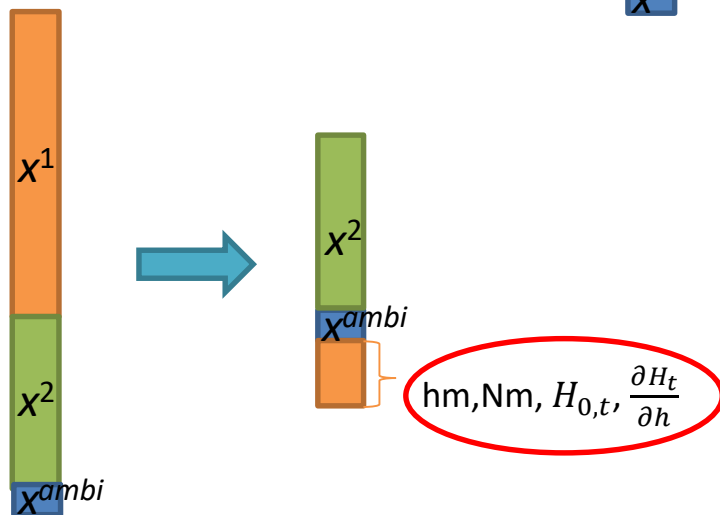
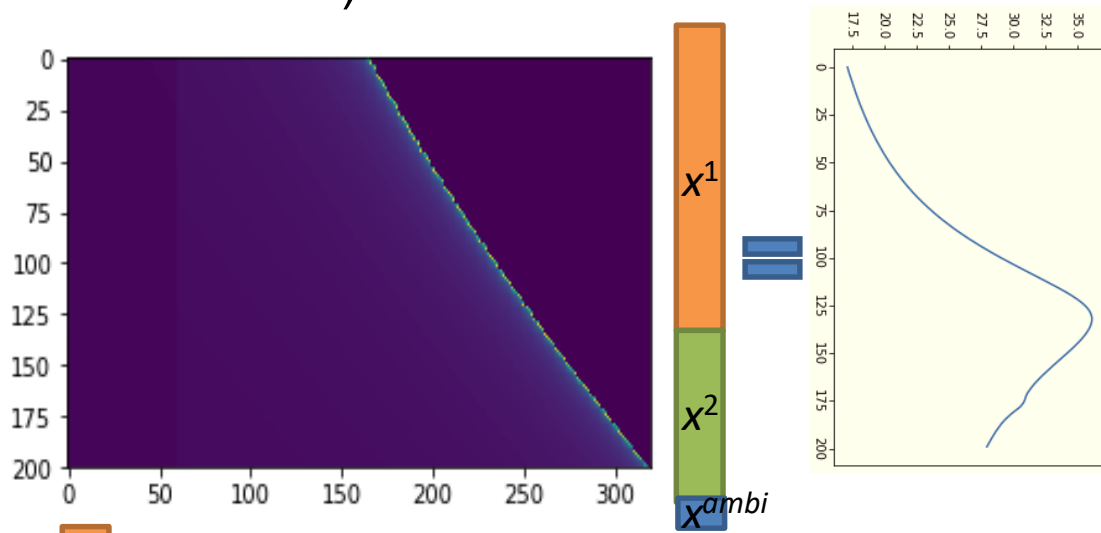


Layer thickness = ~3km

$Ax=b$: 1) rank deficient equations
 2) ill-conditioned equations (column vectors of A matrix are highly related)

3. Method 2: AVHIRO

The unknown vector x in equation $Ax = b$ is composed to three parts x^1 (electron densities from 380 km to 1000 km), x^2 (electron densities below 380 km) and x^{ambi}

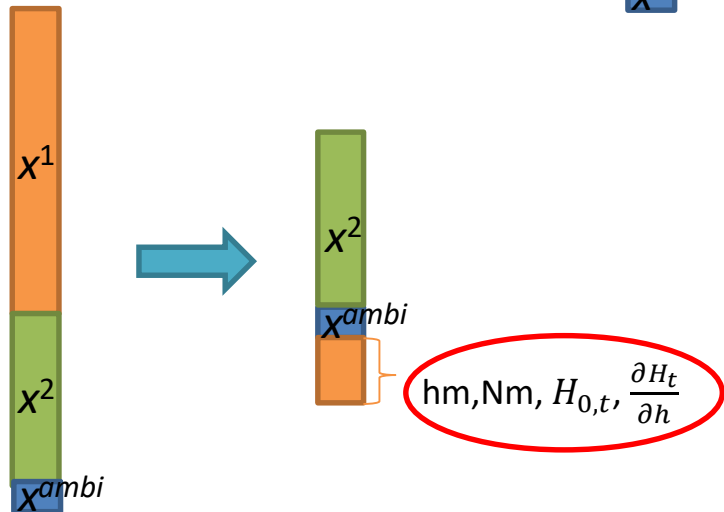
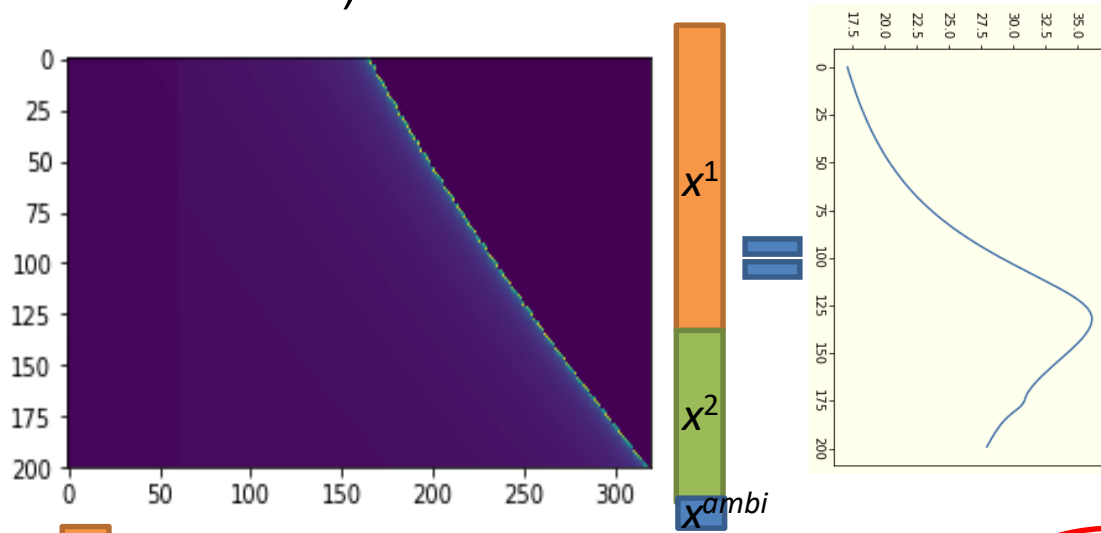


$$N = N_m e^{k(1-z-e^{-z})}, \text{ where } z = \frac{h-h_m}{H}$$

$$H_t(h) = \frac{\partial H_t}{\partial h} (h - h_m) + H_{0,t}, \quad h \geq h_m$$

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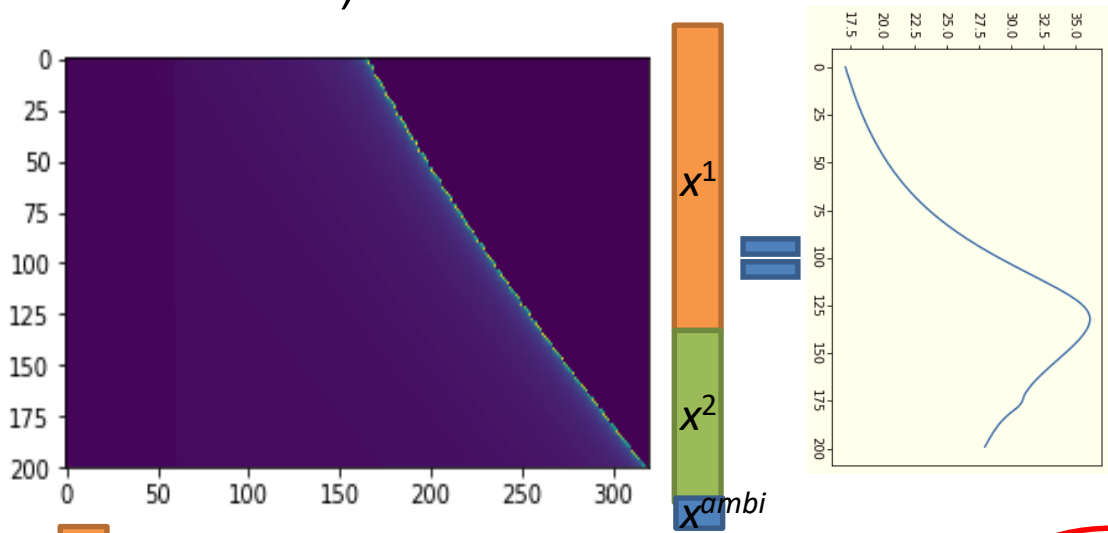
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Nonlinear: two exponential terms

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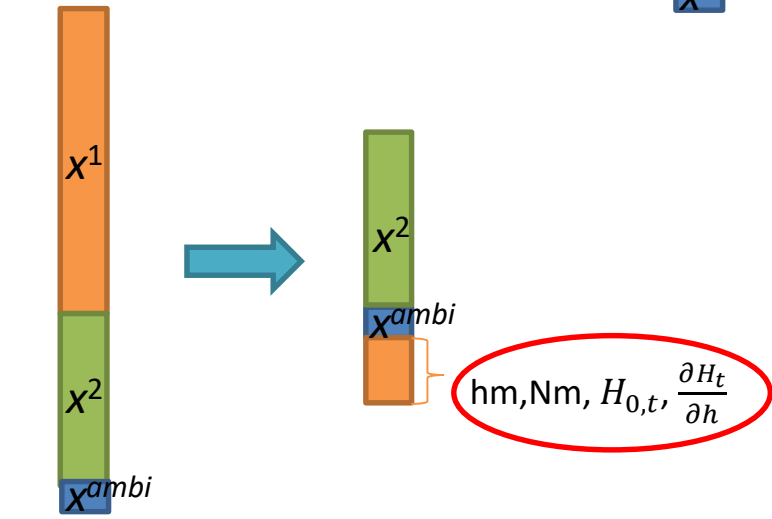


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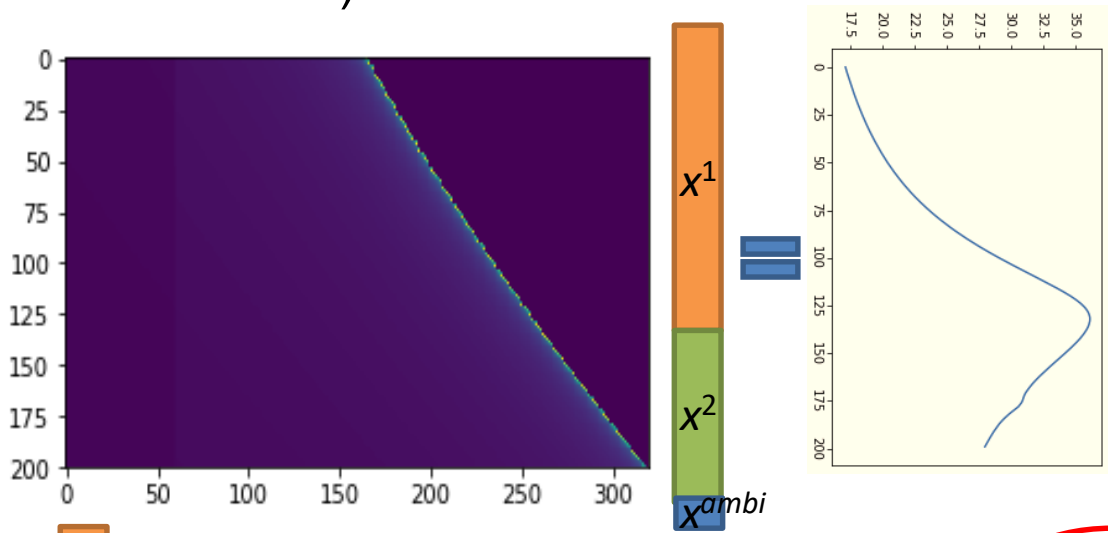
Nonlinear: two exponential terms

Linearization



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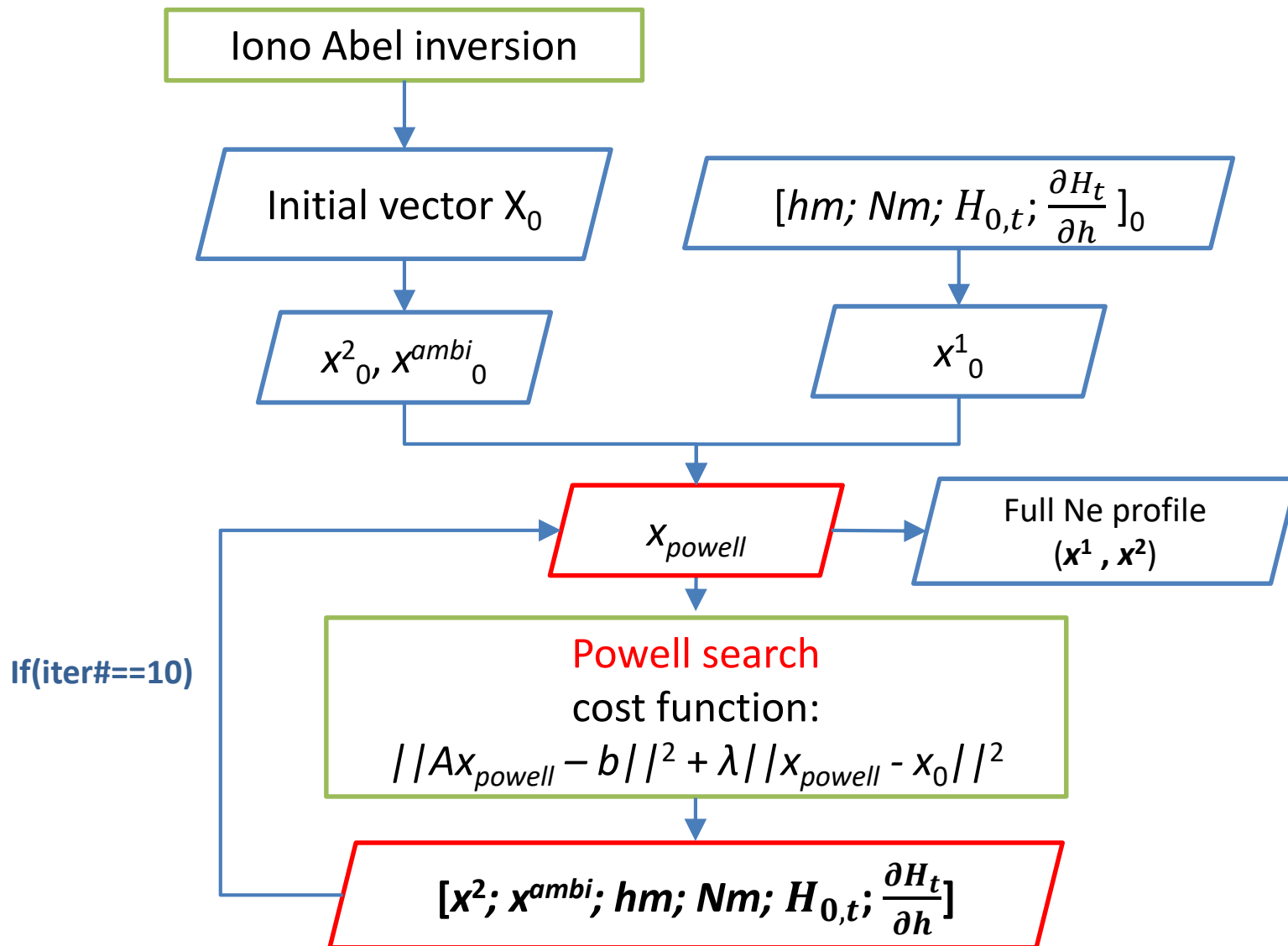
Nonlinear: two exponential terms

Linearization



$$h_m, N_m, H_{0,t}, \frac{\partial H_t}{\partial h}$$

3. Method 2: AVHIRO



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Iono Abel inversion

Initial vector x_0

solve the full electron densities (up to 1000 km), ambiguity term and four parameters of Vary-Chap model *simultaneously*.

x_{powell}

Full Ne profile
(x^1, x^2)

If(iter#==10)

Powell search
cost function:

$$\|Ax_{powell} - b\|^2 + \lambda \|x_{powell} - x_0\|^2$$

$[x^2; x^{ambi}; hm; Nm; H_{0,t}; \frac{\partial H_t}{\partial h}]$

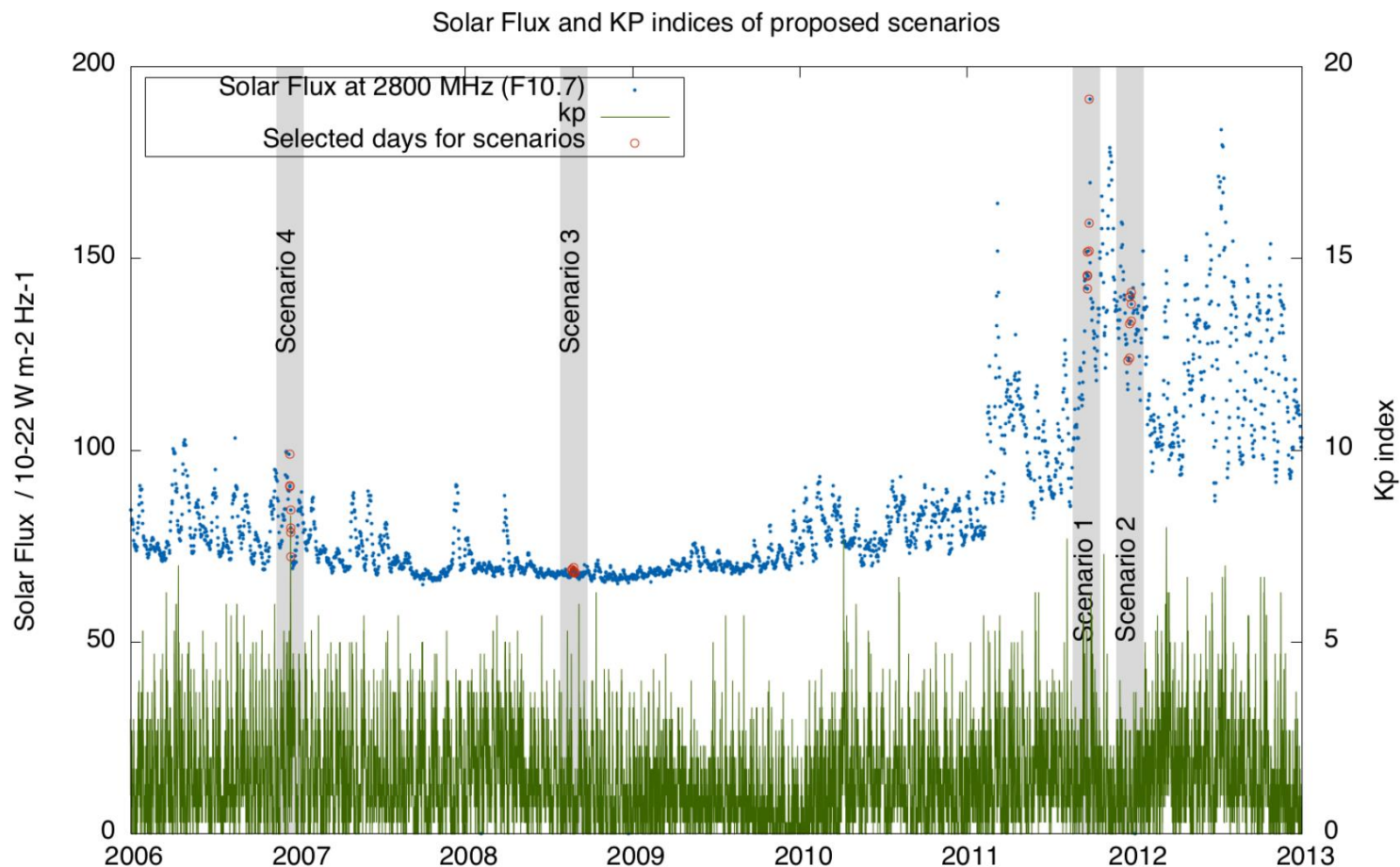
4. Assessment

4.1 SEEIRO assessment (<500km)

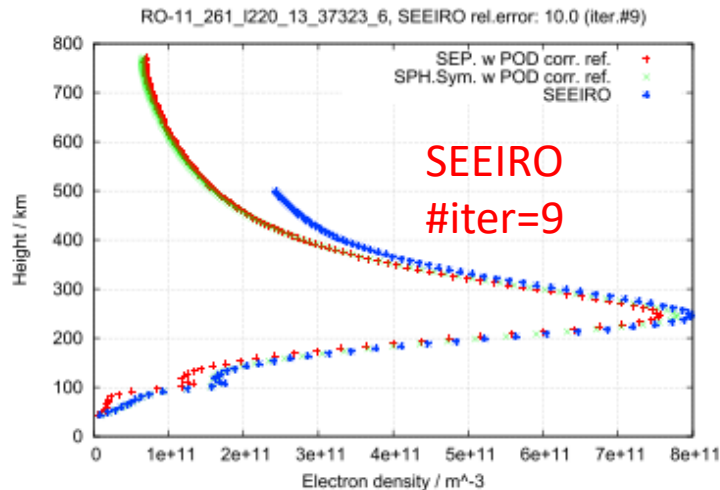
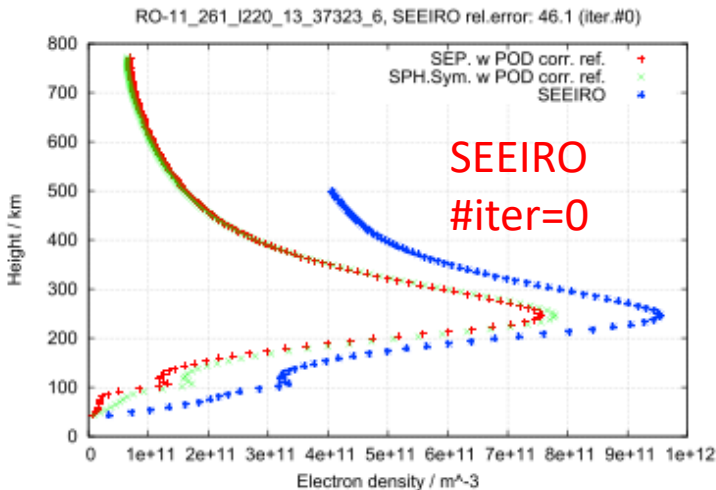
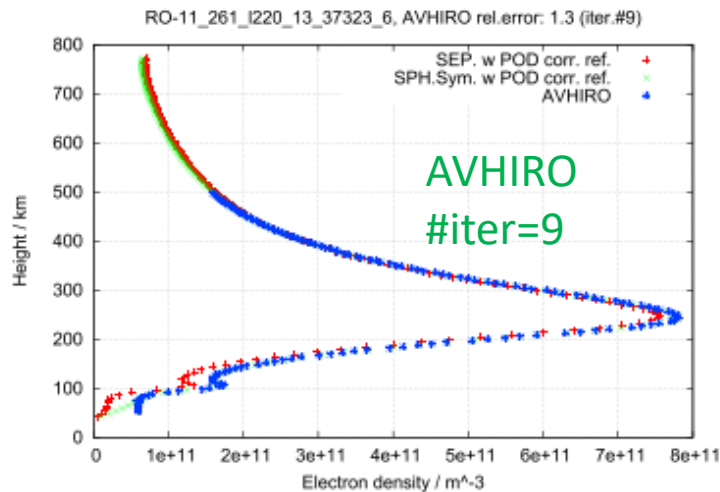
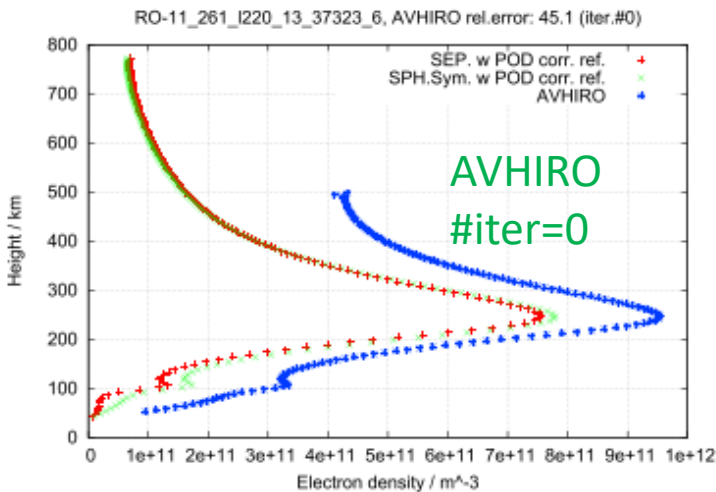
4.2 AVHIRO assessment (<500km)

4.3 Other aspects of assessment regarding AVHIRO

Representative scenarios of major storm, solar cycle minimum and maximum conditions: +3700 COSMIC/FORMOSAT3 radio-occultations (inverted in terms of electron density profiles with improved Abel inversion)



Example for a typical single RO day 261, 2011, ~37323sec, PRN13, rec.I220 (arc#6)



Ref 1: Abel inversion modeling the horizontal variability with separability concept from complete RO data

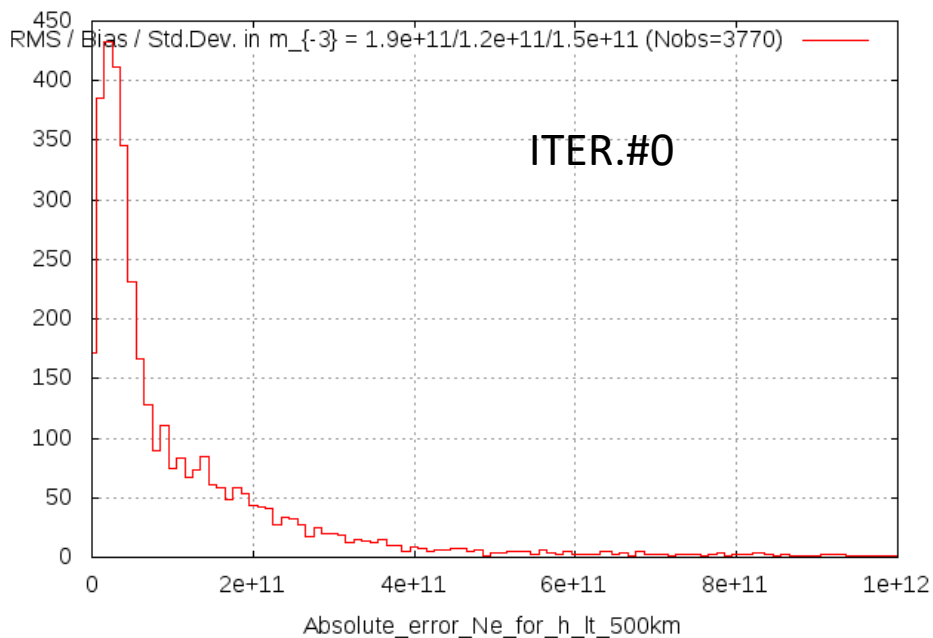
Ref 2: Abel inversion under assumption of spherical symmetry from complete RO data

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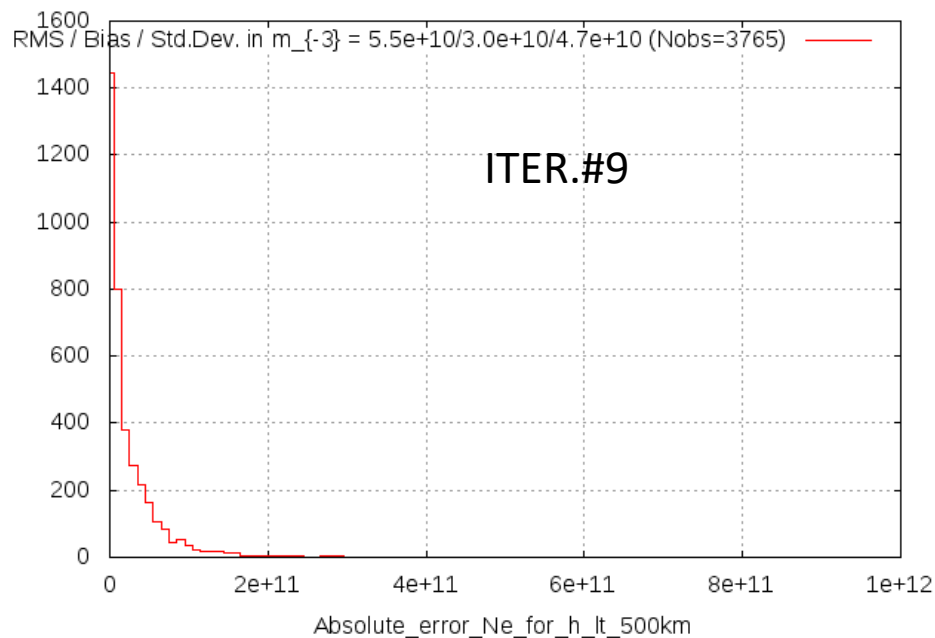
4.1 SEEIRO assessment (<500km)

Distribution of absolute errors (28-days representative data)

COSMIC_FORMOSAT3_4_repr_weeks_ROPE_ITER_0.S2



COSMIC_FORMOSAT3_4_repr_weeks_ROPE_ITER_9.S2



Abs.Error BEFORE SEEIRO:

1.2e+11 m⁻³ +/- 1.5e+11 m⁻³

Abs.Error AFTER SEEIRO:

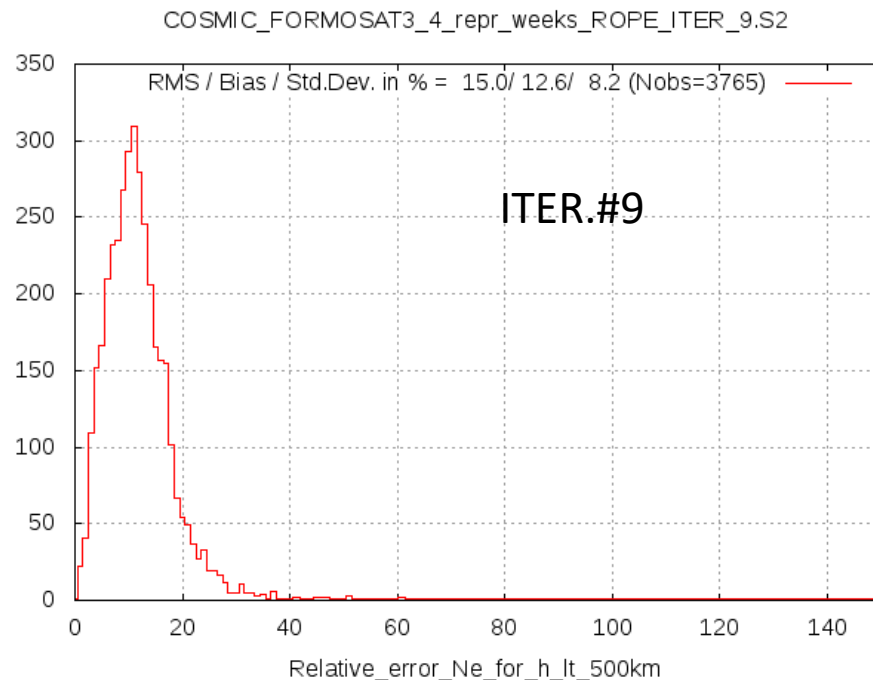
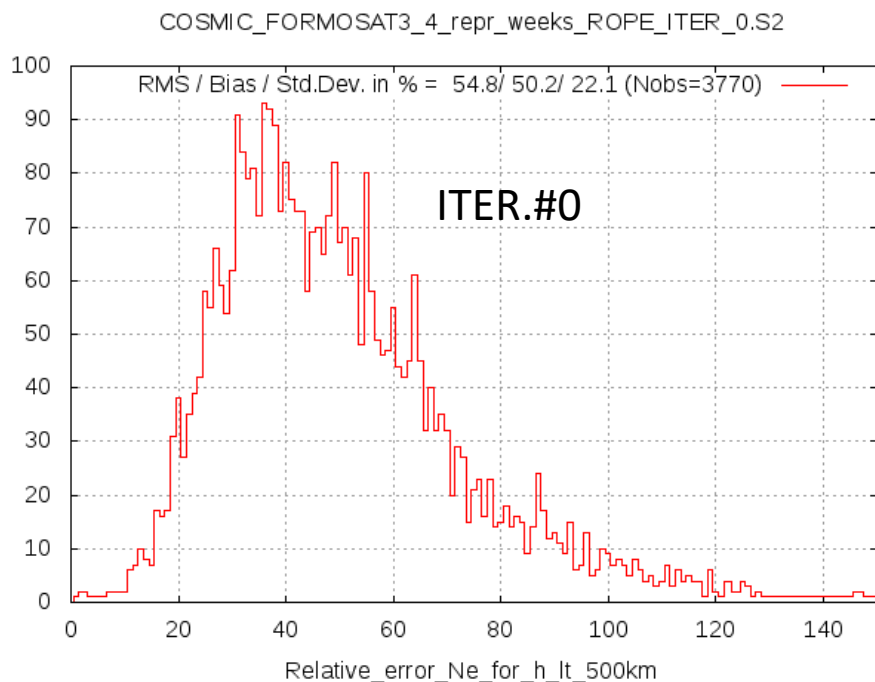
3.0e+10 m⁻³ +/- 4.7e+10 m⁻³

4.1 SEEIRO assessment (<500km)

Distribution of **relative errors**
(28-days representative data)

Each profile:

$$Rel.err = \frac{RMS(\Delta Ne)}{RMS(Ne_{ref})} \times 100\%$$



Rel.Error **BEFORE** SEEIRO:
50% +/- 22%

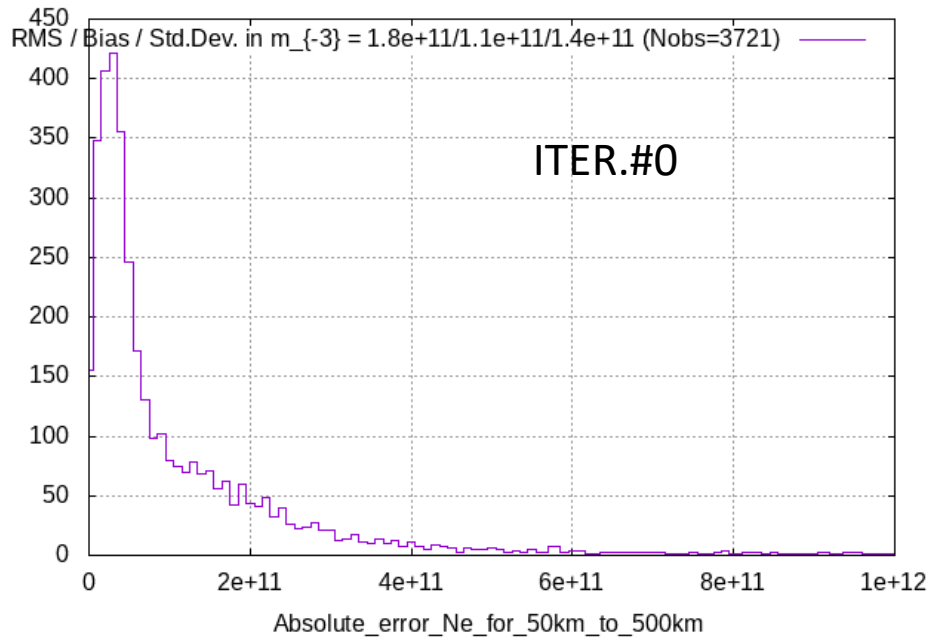
Rel.Error **AFTER** SEEIRO:
13% +/- 8%

4.2 AVHIRO assessment (<500km)

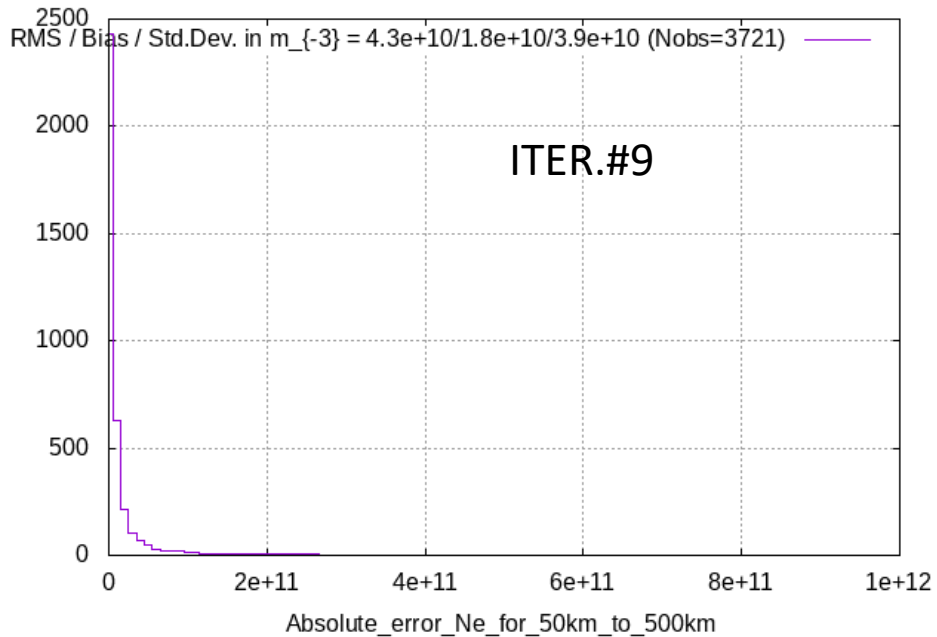
4.2 AVHIRO assessment (<500km)

Distribution of absolute errors (28-days representative data)

COSMIC_FORMOSAT3_4weeks_python-v13_erabs_ini



COSMIC_FORMOSAT3_4weeks_python-v13_erabs



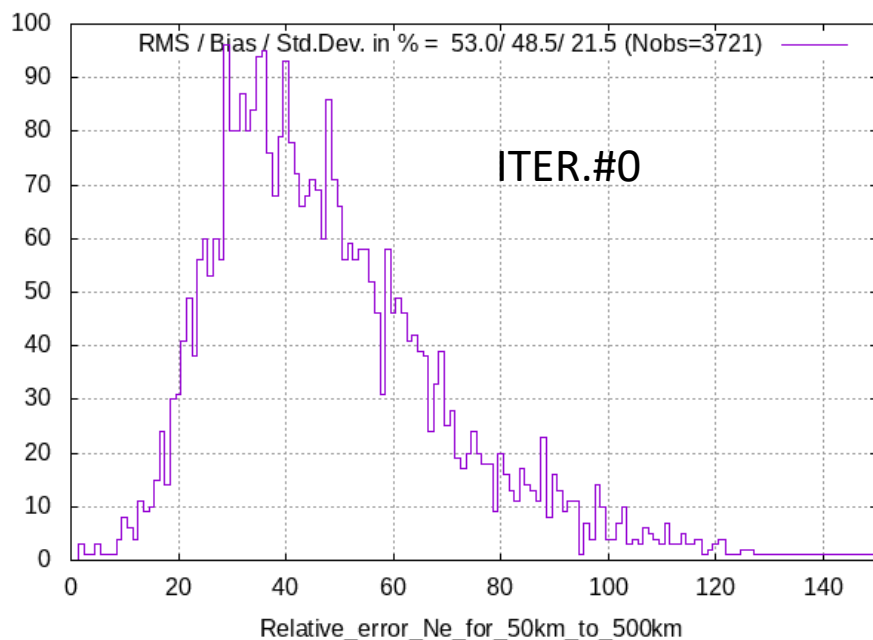
Abs.Error BEFORE AVHIRO:
1.1e+11 m⁻³ +/- 1.4e+11 m⁻³

Abs.Error AFTER AVHIRO:
1.8e+10 m⁻³ +/- 3.9e+10 m⁻³

4.2 AVHIRO assessment (<500km)

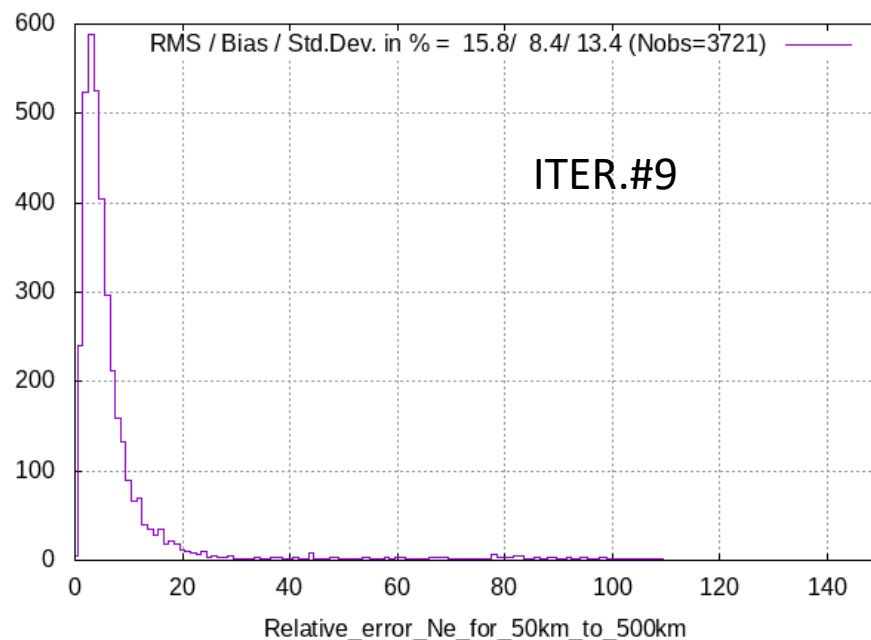
Distribution of **relative errors** (28-days representative data)

COSMIC_FORMOSAT3_4weeks_python-v13_errel_ini



Rel.Error **BEFORE** AVHIRO:
49% +/- 22%

COSMIC_FORMOSAT3_4weeks_python-v13_errel

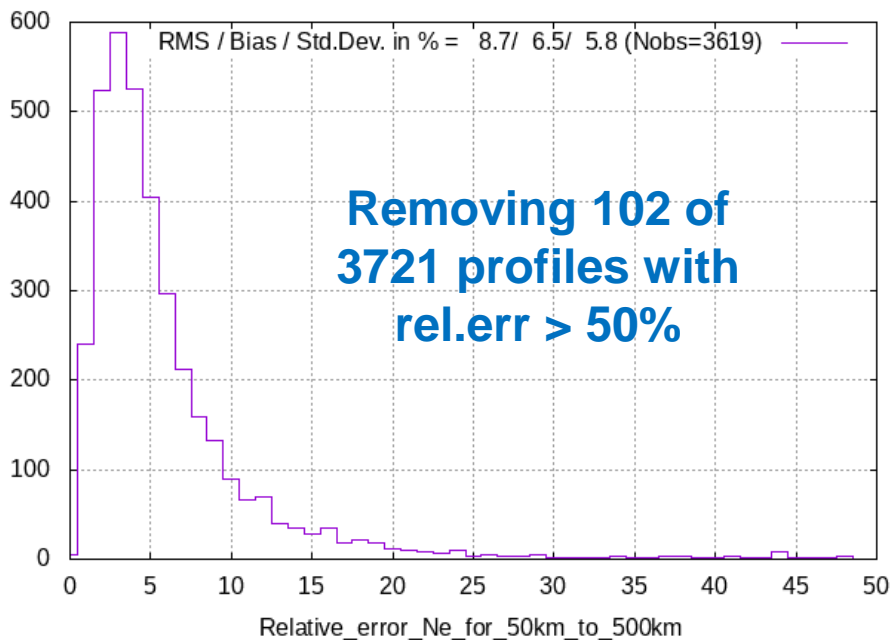


Rel.Error **AFTER** AVHIRO:
8% +/- 13%

4.2 AVHIRO assessment (<500km)

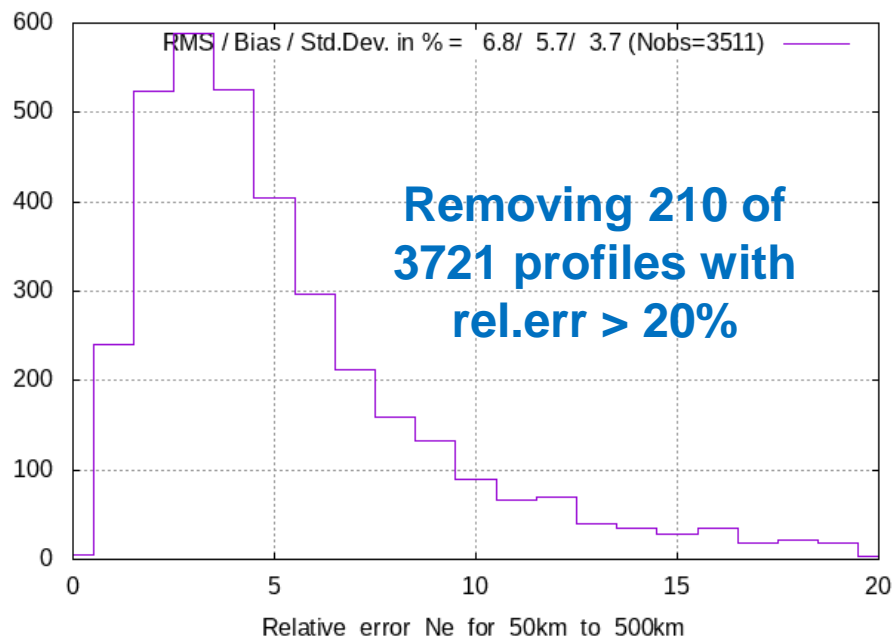
Distribution of **relative errors** (28-days representative data)

COSMIC_FORMOSAT3_4weeks_python-v13_errel_no_outliers_greater_than_50



Rel.Error AFTER AVHIRO:
7% +/- 6%

COSMIC_FORMOSAT3_4weeks_python-v13_errel_no_outliers_greater_than_20



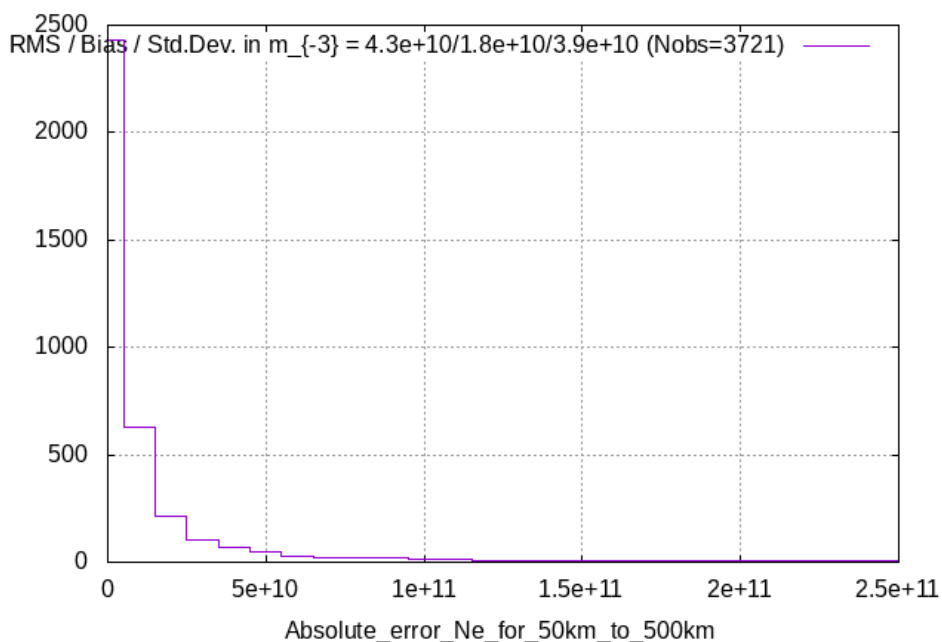
Rel.Error AFTER AVHIRO:
6% +/- 4%

4.3 Other aspects of assessment regarding AVHIRO

Absolute error below and above 500 km

Below 500 km

COSMIC_FORMOSAT3_4weeks_python-v13_erabs

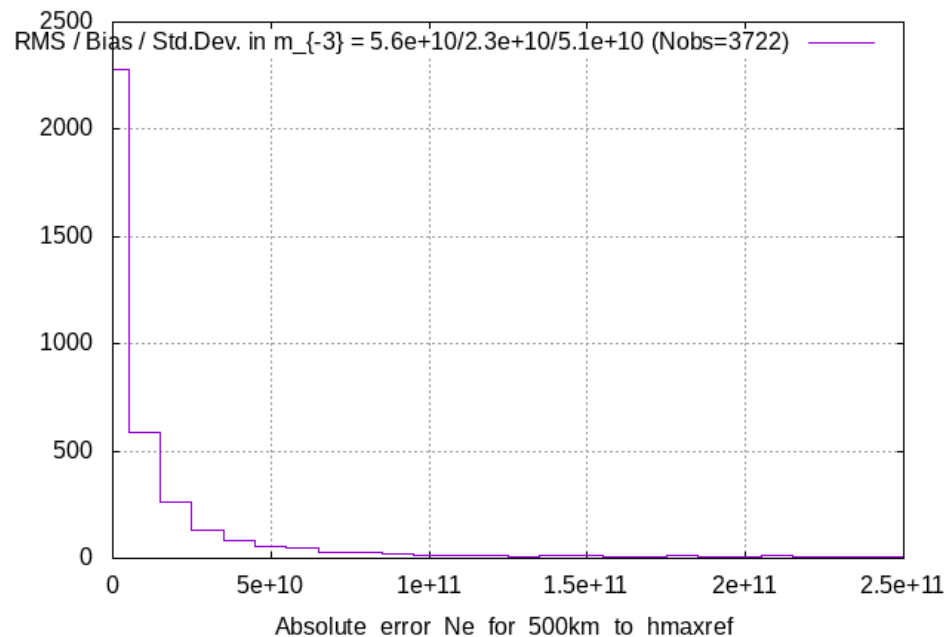


Abs.Error:

$1.8e+10 \text{ m}^{-3} \pm 3.9e+10 \text{ m}^{-3}$

Blind area: above 500 km

COSMIC_FORMOSAT3_4weeks_python-v13_erabs



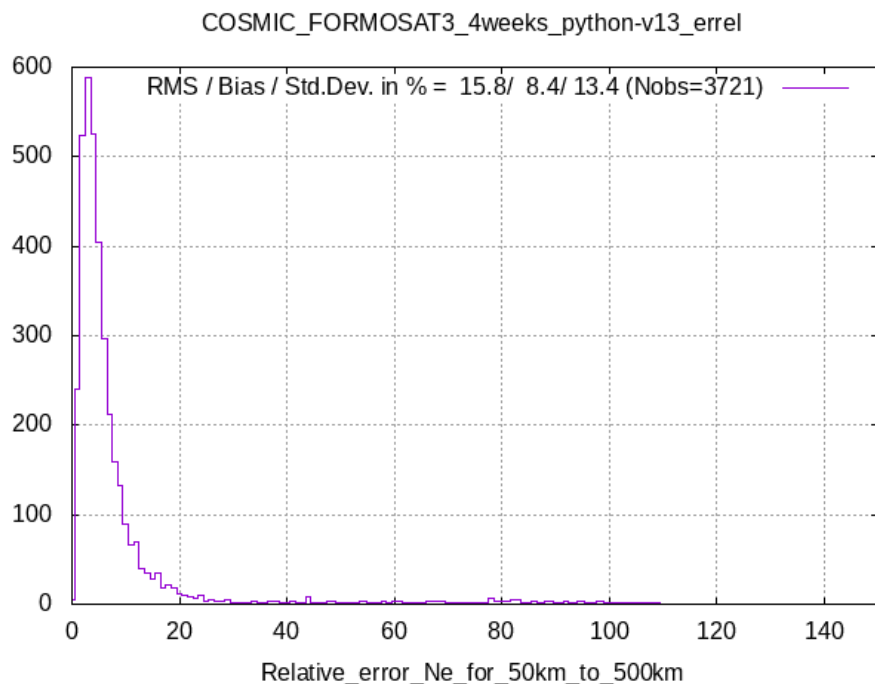
Abs.Error:

$2.3e+10 \text{ m}^{-3} \pm 5.1e+10 \text{ m}^{-3}$

Relative error below and above 500 km

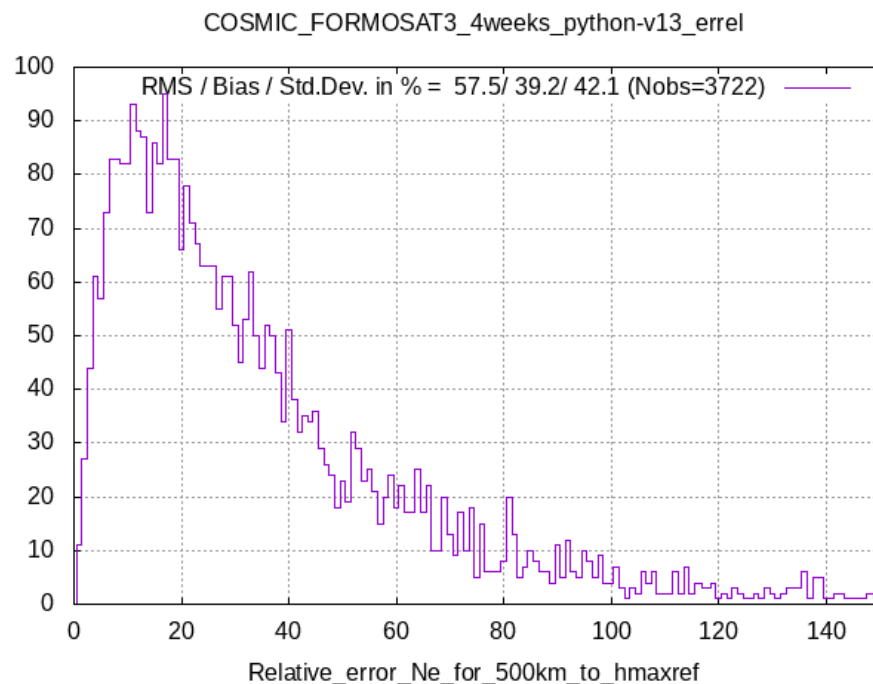
$$\text{Each profile: Rel.err} = \frac{RMS(\Delta Ne)}{RMS(Ne_{ref})} \times 100\%$$

Below 500 km



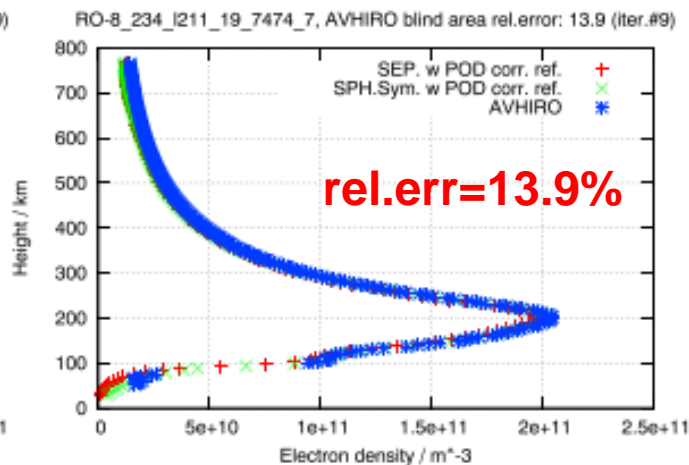
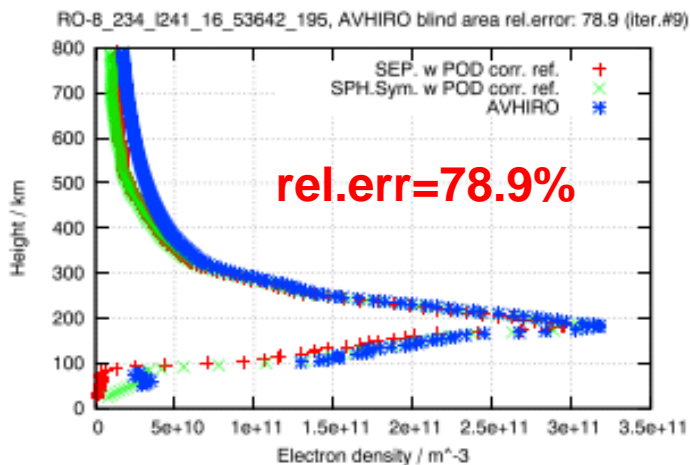
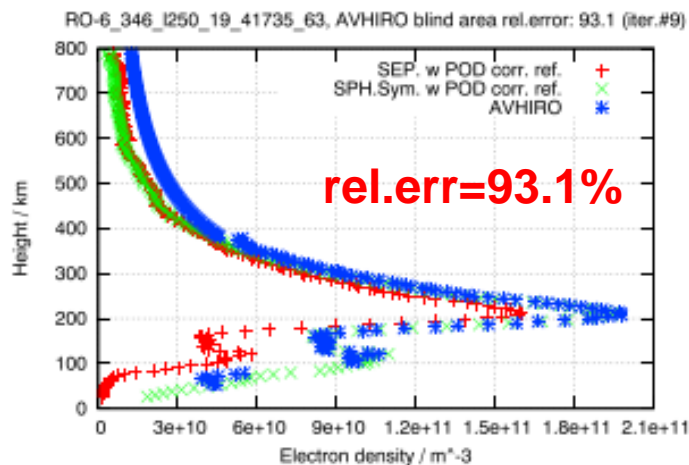
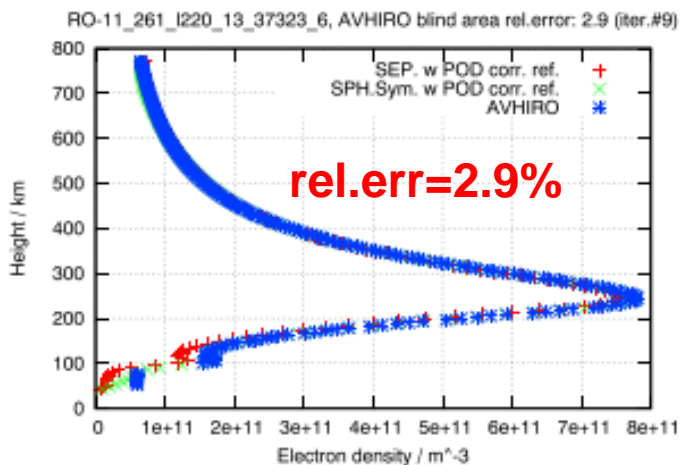
Rel.Error:
8% +/- 13%

Blind area: above 500 km



Rel.Error:
39% +/- 42%

Four examples of precision above 500 km



Ref 1: Abel inversion modeling the horizontal variability with separability concept from complete RO data

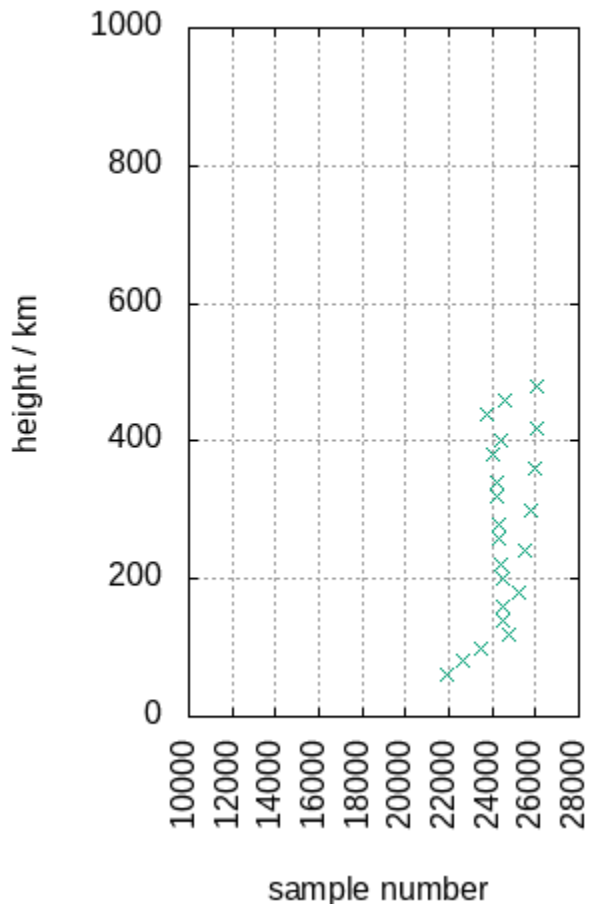
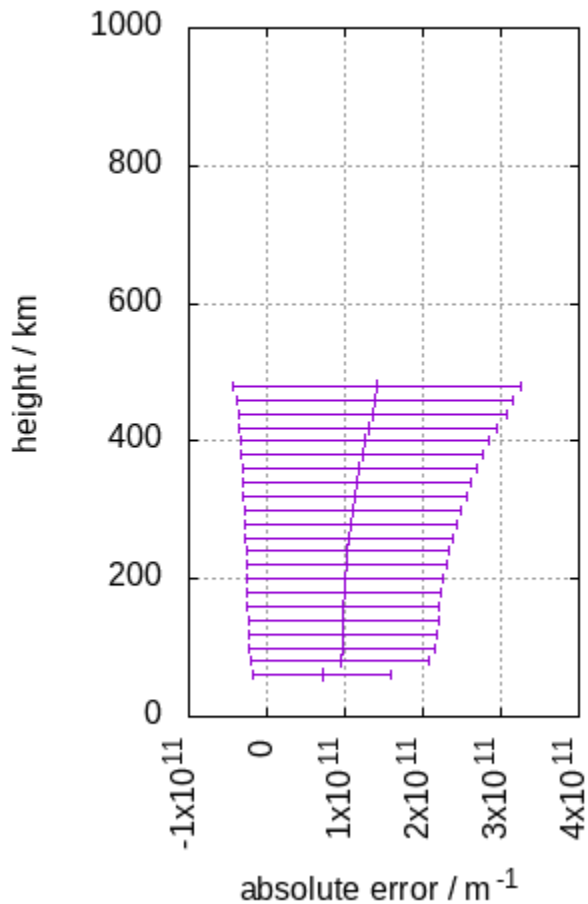
Ref 2: Abel inversion under assumption of spherical symmetry from complete RO data

AVHIRO: Abel-VaryChap Hybrid density profile from topside Incomplete RO data

Absolute Error vs. height

ITER.#0

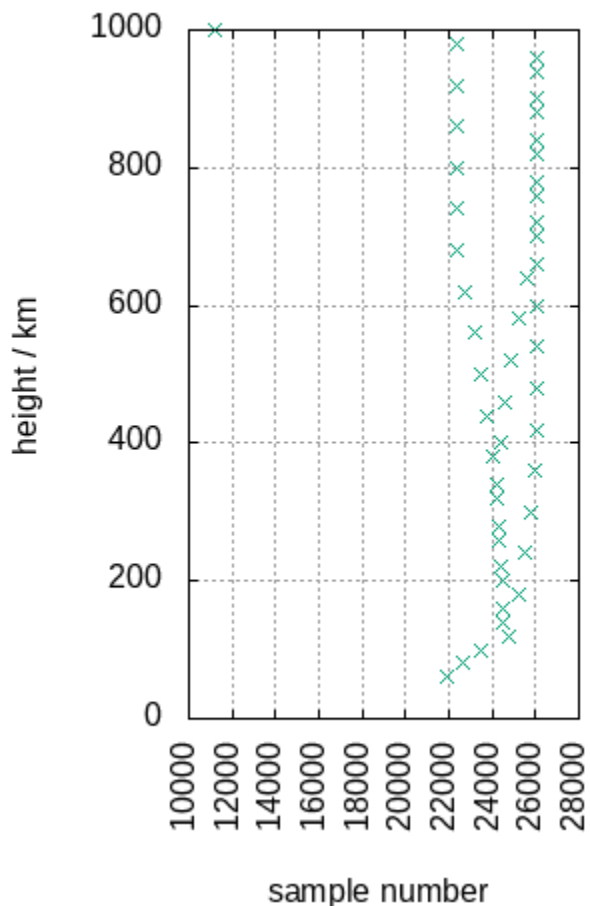
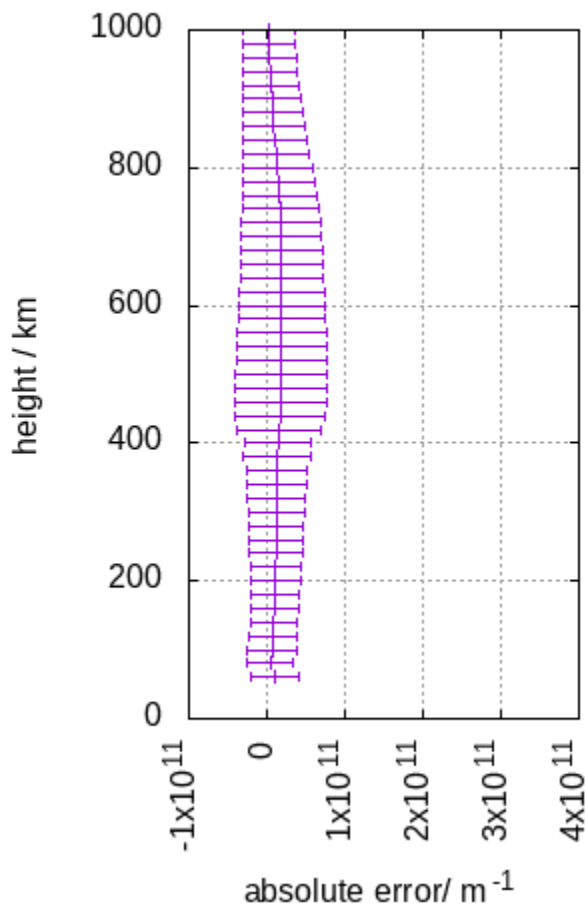
initial solution



Absolute Error vs. height

ITER.#9

final solution



Pros and Cons: Summary

	AVHIRO	SEEIRO
Ne Relative Accuracy	8%	13%
Predominant Ne Rel. Acc.	3%	10%
Ne Absolute Accuracy	$(1.8 \times 10^{10} \pm 3.9 \times 10^{10}) \text{m}^{-3}$	$(3.0 \times 10^{10} \pm 4.7 \times 10^{10}) \text{m}^{-3}$
Predominant Ne Abs. Acc.	$< 10^{10} \text{m}^{-3}$	$< 10^{10} \text{m}^{-3}$
CPU time per preprocessed RO	20 minutes (~2min in future parallel version)	17 seconds
Suitable for NRT service?	No	Yes
Suitable for PP service?	Yes	Backup option for PP
Required ancillary information?	No	No
Required inputs	Dual-frequency GPS carrier phase measurements, predicted GPS and LEO orbits	
Convenient inputs	Dual-frequency GPS POD carrier phase meas.	

5. Conclusions (1/2)

1. In this work, two methods - SEEIRO and AVHIRO - are presented as new techniques to retrieve full electron density profile from truncated RO data, **without the need of external data**.
2. SEEIRO reduces the relative error of the RO inversion with data up to 500 km regarding to the full data inversion up to 800 km, from **50% (+/-22%) before, to 13%(+/-8%)**
3. AVHIRO reduces the electron density error of the RO inversion with measurements up to 500 km regarding to the full inversion with observations up to 800 km: from **49% (+/-22%) before to 8%(+/-13%)**.
4. **AVHIRO** reduces the predominant relative error to **3%** compared with the **10%** obtained with the fast **SEEIRO** approach.

5. Conclusions (2/2)

5. **AVHIRO** provides simultaneously the linear Vary-Chapman extrapolated electron density profile with accuracy **slightly lower than** those obtained at heights below 500 km with observations: **$(2.3 \pm 5.1) \times 10^{10} \text{ m}^{-3}$** above versus **$(1.8 \pm 3.9) \times 10^{10} \text{ m}^{-3}$** below 500 km.
6. **SEEIRO** solves the problem by two step processing strategy and needs less computation time, suitable for **Near Real-Time** determination. **AVHIRO** estimates the full electron density profile simultaneously and achieves more accurate results, yet at higher computational cost, suited for **postprocessing**.

Remarks

In the proof of concept of SEEIRO and AVHIRO, spherical symmetry is assumed. The improved Abel inversion taking into account horizontal gradients will be easily applied in a straightforward way under separability hypothesis by substituting $N_i = V_i * S_i$, where V_i is the VTEC at the corresponding "i" crossing point (given for instance by GIM) and being S_i , the "shape function", the new unknown.

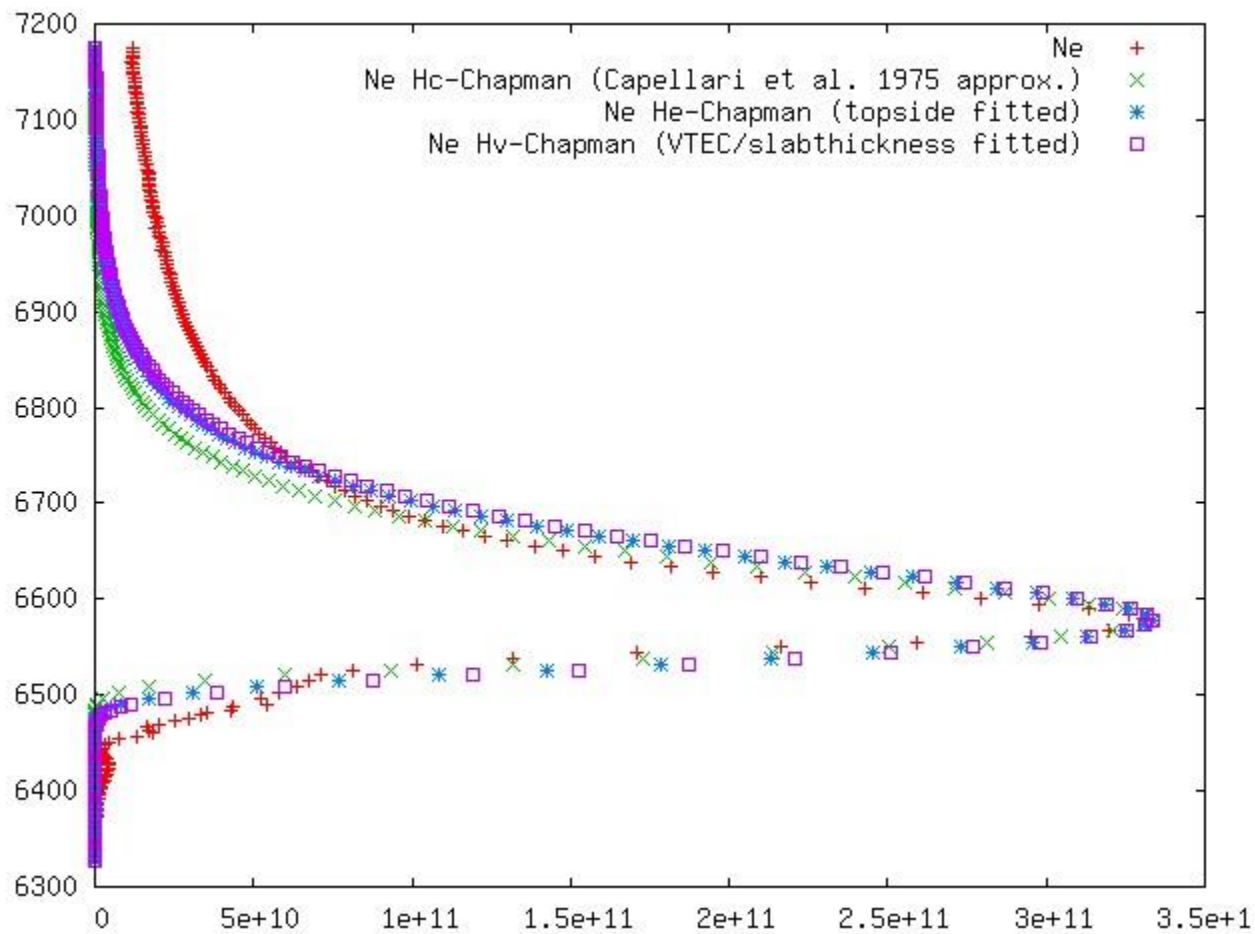
Reference

1. Olivares-Pulido, G., Hernández-Pajares, M., Aragón-Àngel, A., & García-Rigo, A. (2016). A linear scale height Chapman model supported by GNSS occultation measurements. *Journal of Geophysical Research: Space Physics*, *121*, 7932–7940. <https://doi.org/10.1002/2016JA022337>
2. Hernández-Pajares, M., Garcia-Fernández, M., Rius, A., Notarpietro, R., von Engel, A., Olivares-Pulido, G., et al. (2017). Electron density extrapolation above F2 peak by the linear Vary-Chap model supporting new Global Navigation Satellite Systems-LEO occultation missions. *Journal of Geophysical Research: Space Physics*, *122*, 9003–9014. <https://doi.org/10.1002/2017JA023876>
3. Prol, F. d. S., Themens, D. R., Hernández-Pajares, M., Camargo, P. d. O., & Muella, M. T. d. A. H. (2019). Linear Vary-Chap topside electron density model with topside sounder and radio-occultation data. *Surveys in Geophysics*, *40*, 1–17. <https://doi.org/10.1007/s10712-019-09521-3>
4. Lyu, H., Hernández-Pajares, M., Monte-Moreno, E., & Cardellach, E. (2019). Electron density retrieval from truncated Radio Occultation GNSS data. *Journal of Geophysical Research: Space Physics*, *124*(6), 4842–4851.

Thank you for your attention !

Backup 1

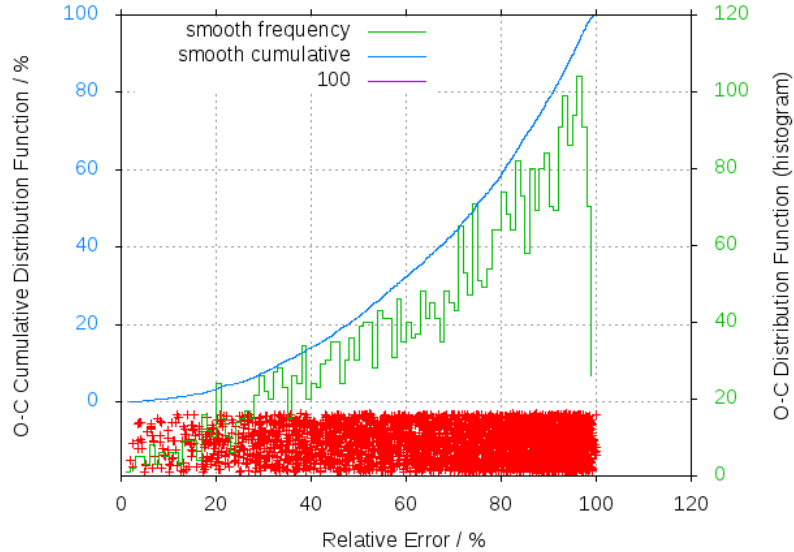
The Vary-Chapman Extrapolation Technique (VCET)



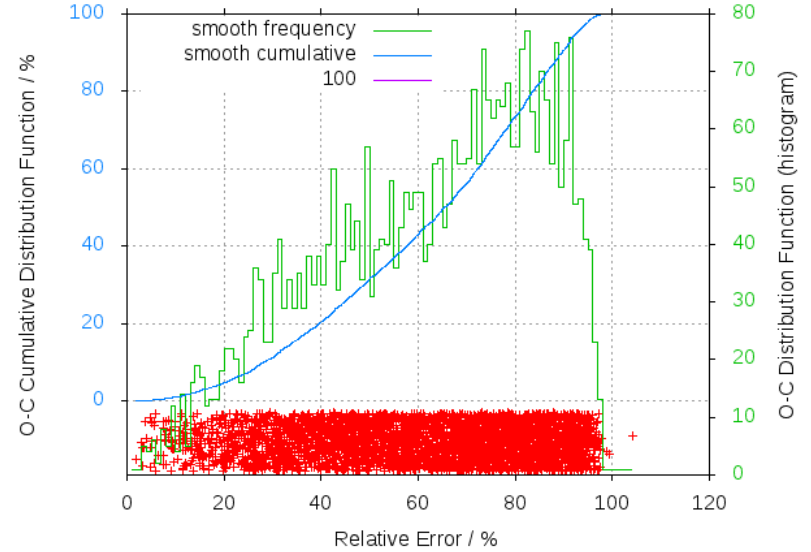
Backup 2

Summary of results on the +3700 occultations in the 4 scenarios

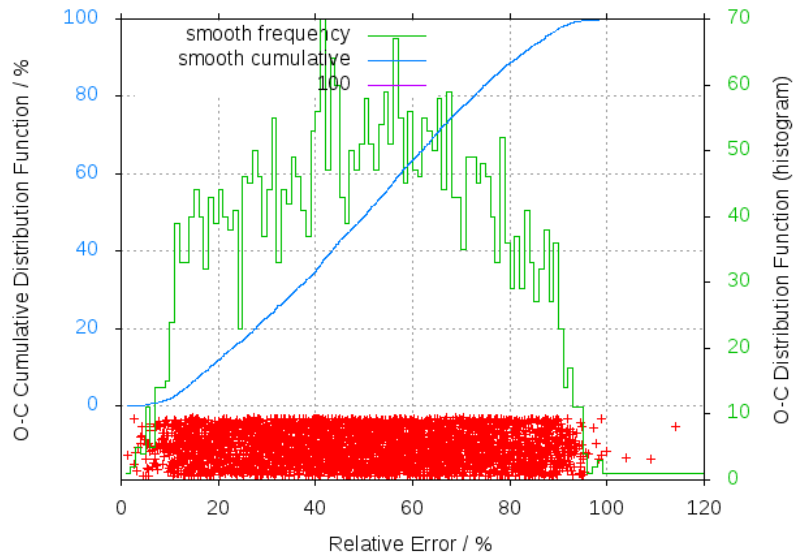
err_rel_Capellari



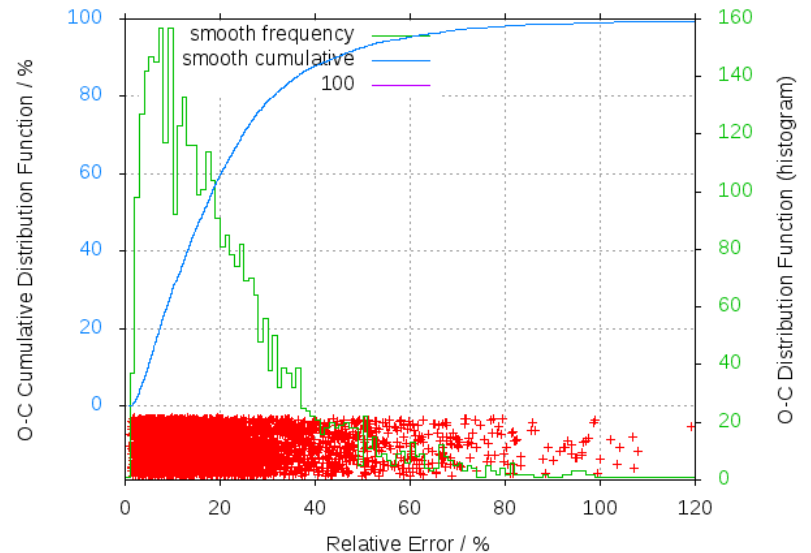
err_rel_Hiter_topaver



err_rel_Hvtec_abel



err_rel_Vary-Chapman



Backup 3

Convergence for ROs of a single day (261, 2011): SEEIRO rel. error vs iteration

